Monopoles, Anapoles, Skyrmions, Majoranas and Higgs
Welcome to the New World of Condensed Matter Physics

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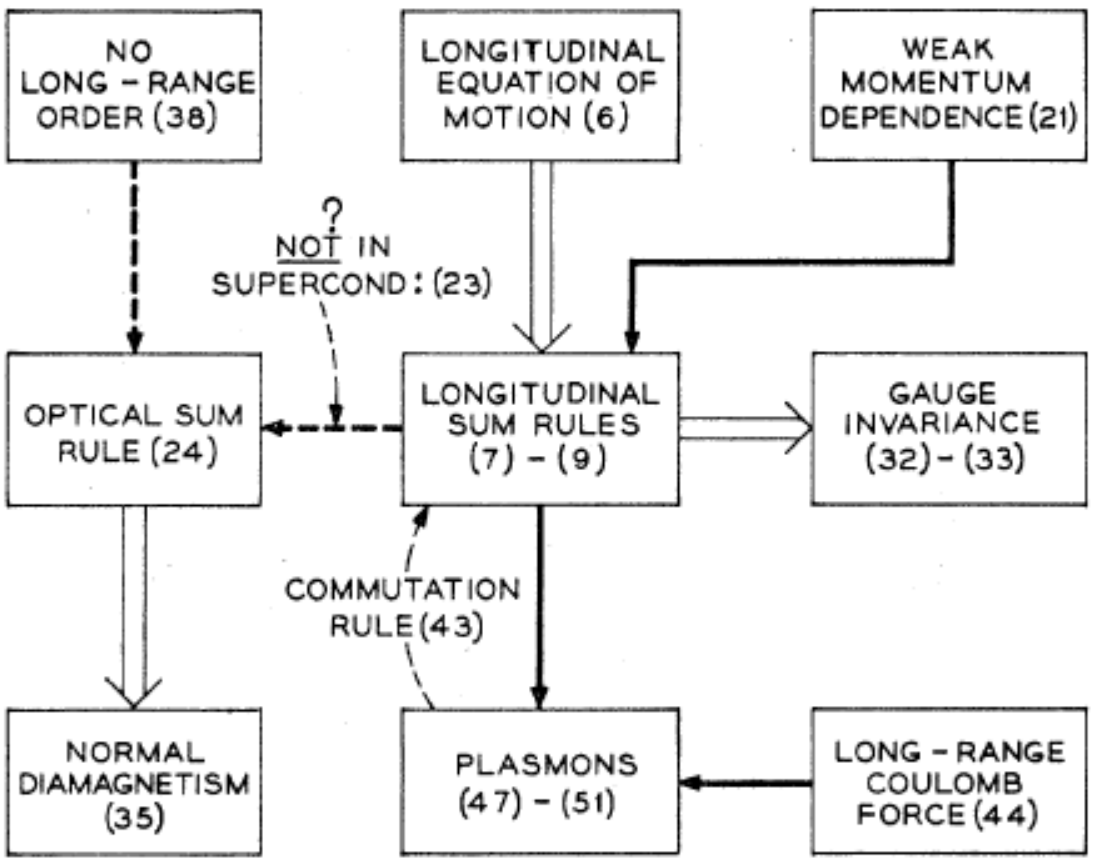
ANL – Jan. 17, 2014

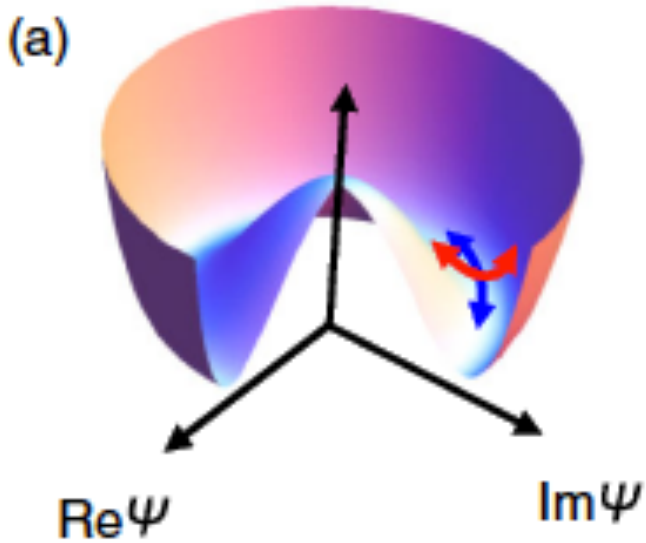
Meissner Effect → London → GL (ϕ^4) → BCS → Anderson

Pair formation causes a 2Δ gap in the transverse EM response
Longitudinal EM response is already gapped (plasmons)
Goldstone (phase) mode is "eaten" by the plasmons



Anderson, Phys Rev (1958 & 1963)

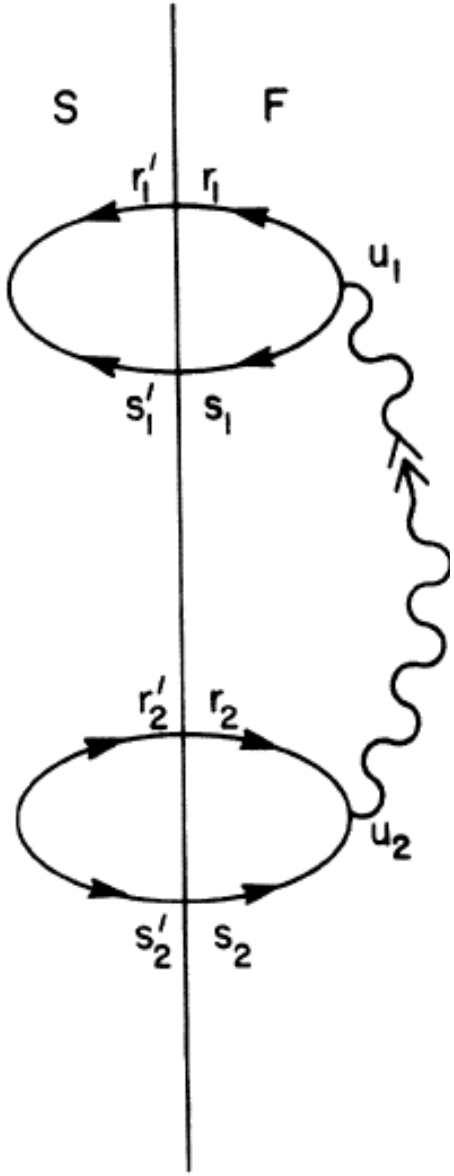
Amplitude `Higgs' (Schmid) Mode

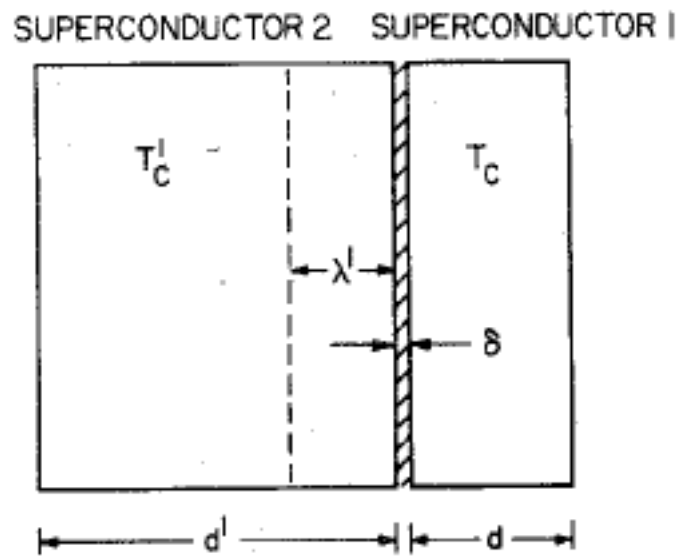


[7] One may explain this mode as follows: If a Cooper pair were to dissociate spontaneously, it would lack the binding energy 2Δ in the dissociated state. Hence, by the incertainty relation, it may remain in this state only for a time $\sim \frac{1}{2\Delta}$. This means that it is forced to oscillate back and forth.

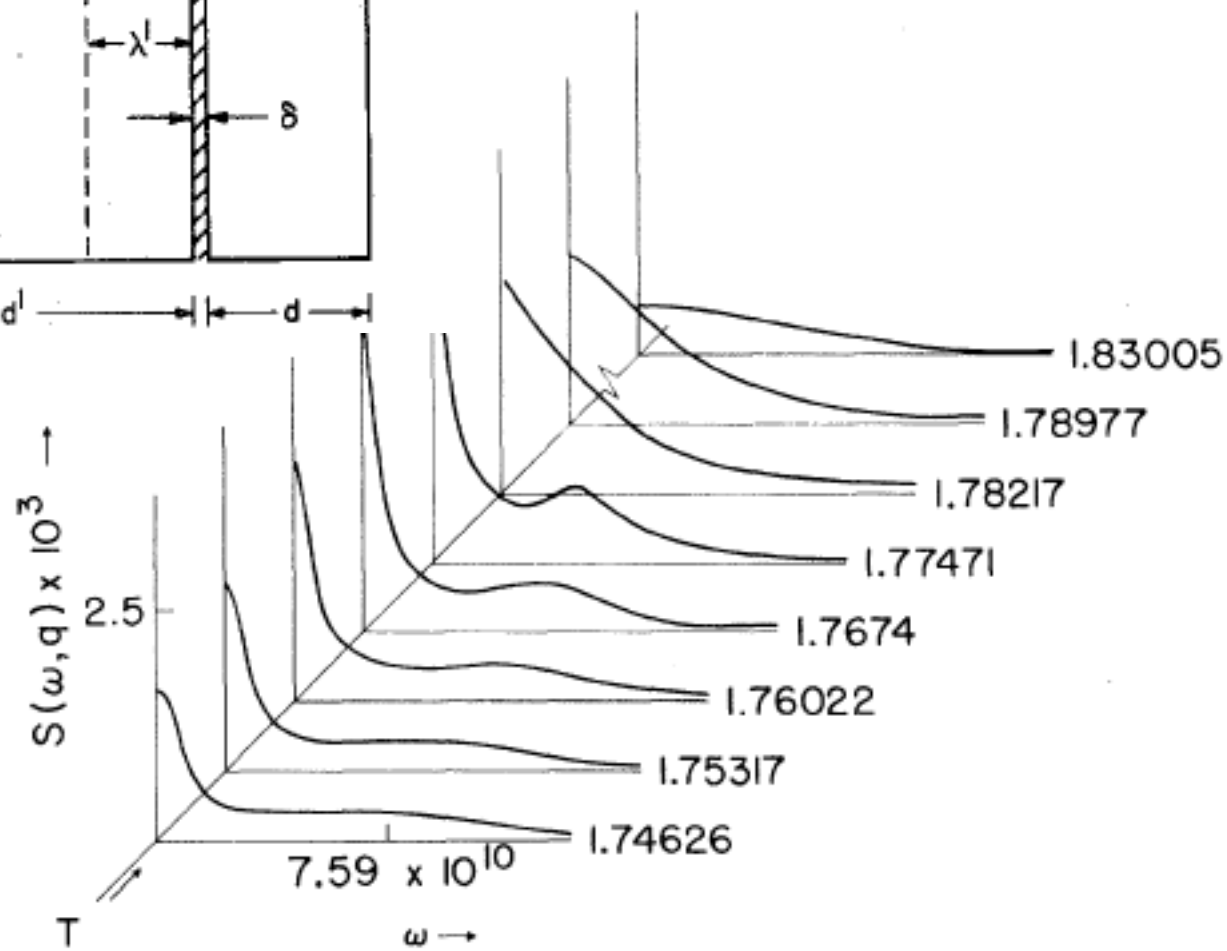
Schmid, Phys. Kondens. Materie (1968)

Carlson-Goldman "phase" mode detected by excess current





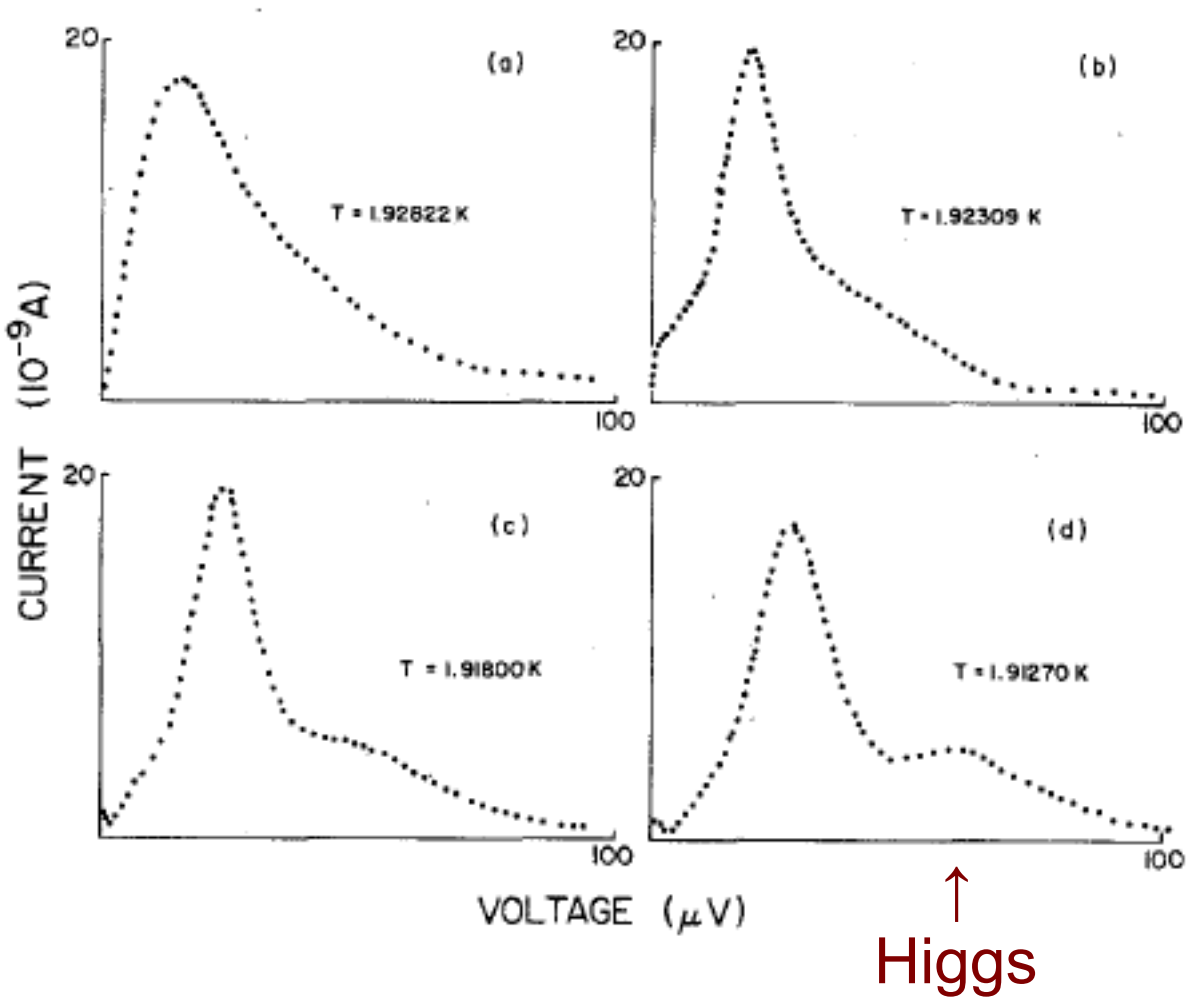
$I(V,H) \propto \mathrm{Im}\,\chi(q,\omega)$



Ferrrell, J LTP (1969)
Scalapino, PRL (1970)
Shenoy & Lee, PRB (1974)

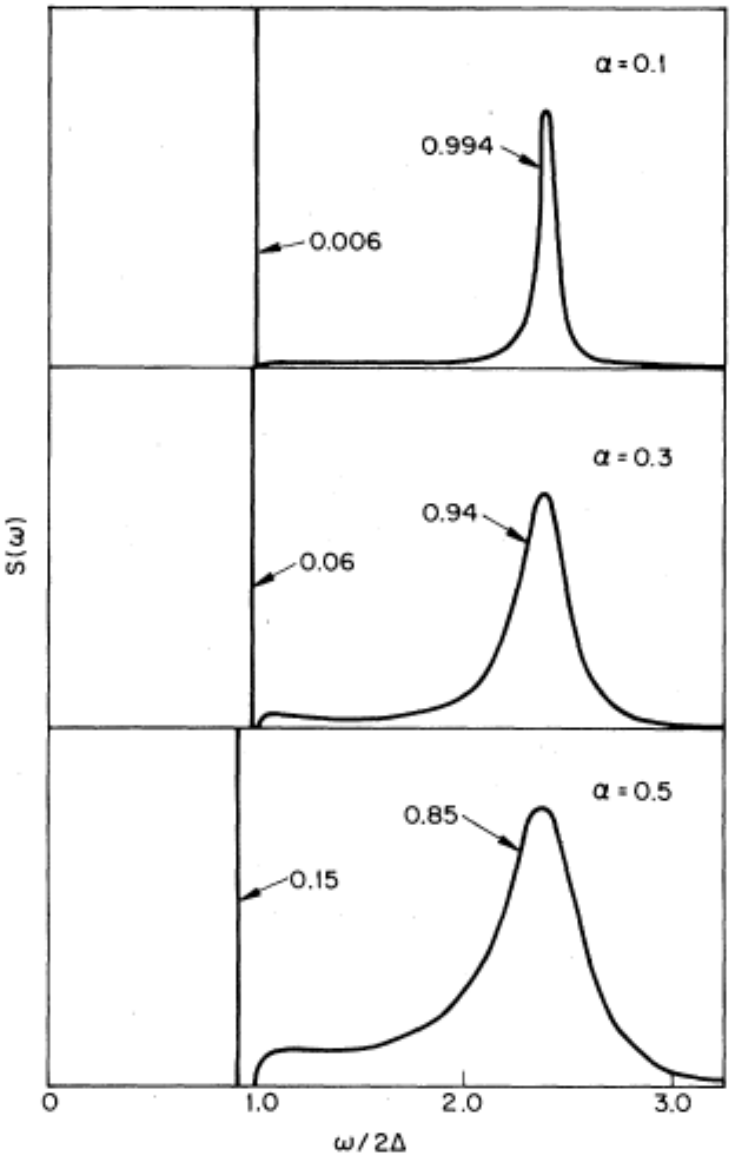
Carlson & Goldman, PRL (1974)
Schmid & Schon, PRL (1975)

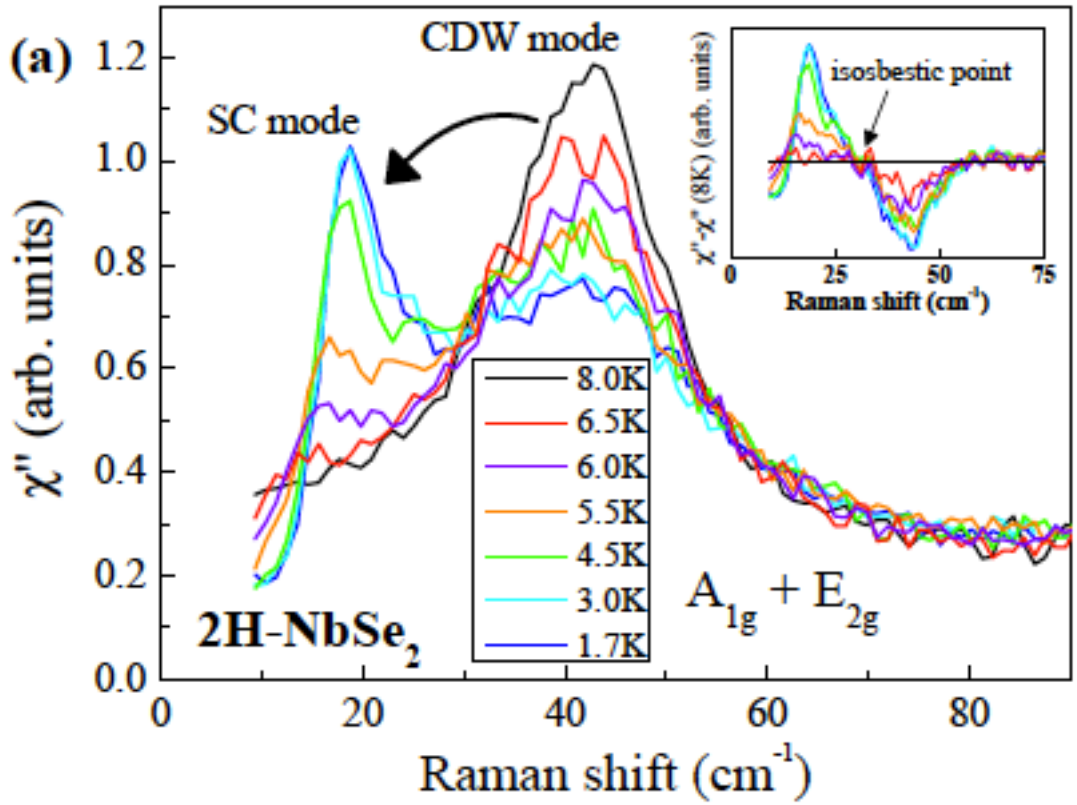
Secondary 2Δ peak (`Higgs' mode) seen but not recognized as such



Carlson & Goldman, J LTP (1976)

Transfer of weight from CDW Higgs mode to SC Higgs mode

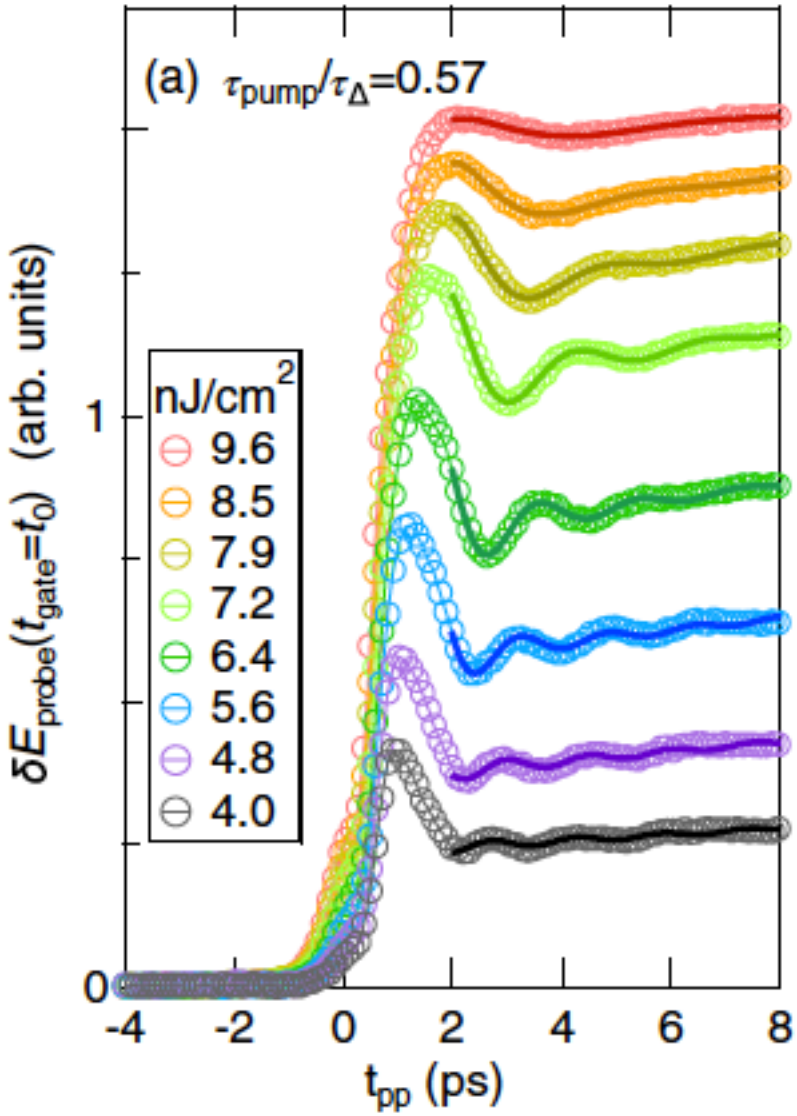




Littlewood & Varma, PRL (1981)

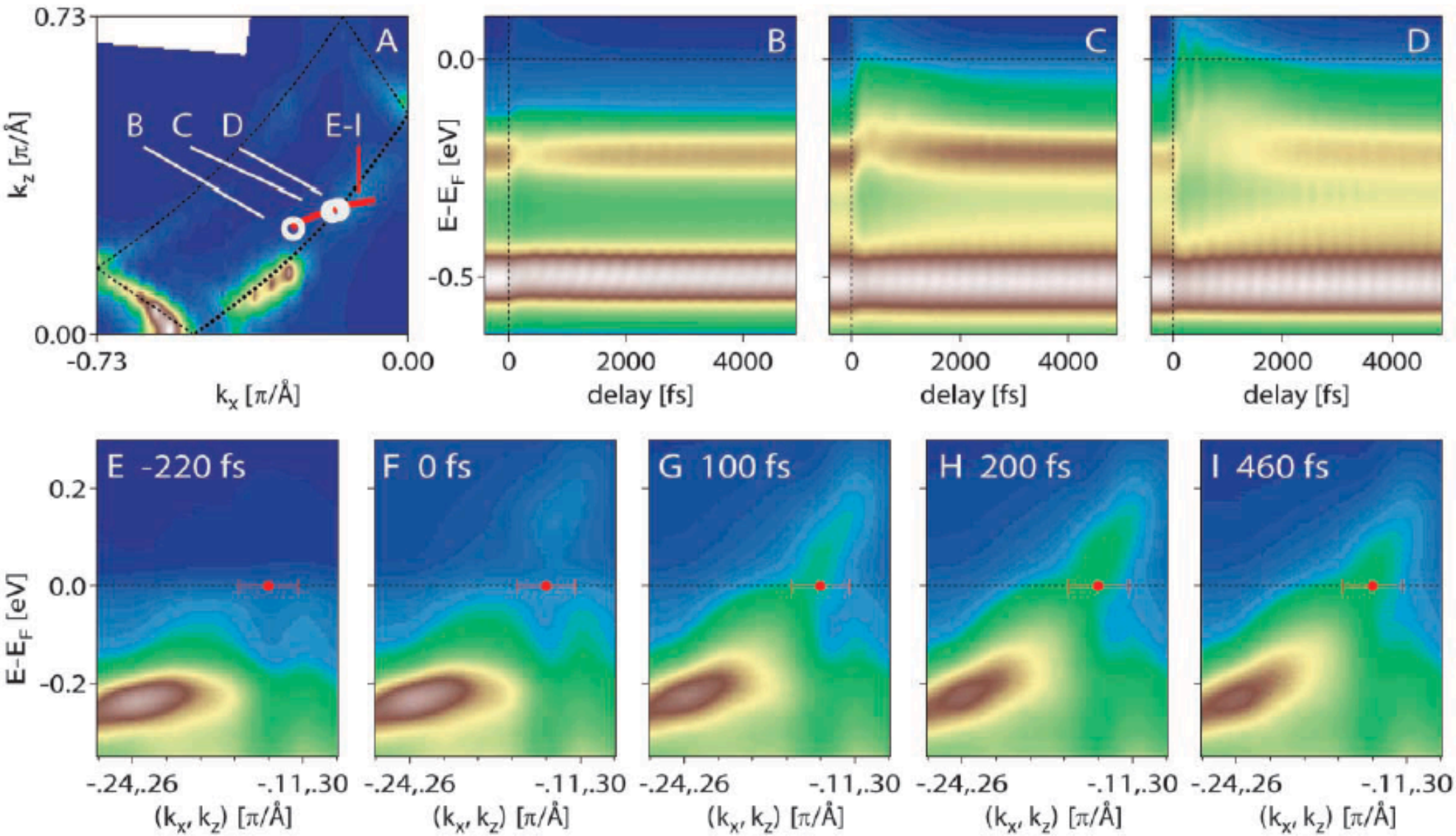
Sooryakumar & Klein, PRL (1980)
Measson *et al*, arXiv (2013)

Higgs oscillations seen in THz pump-probe experiments of NbTiN



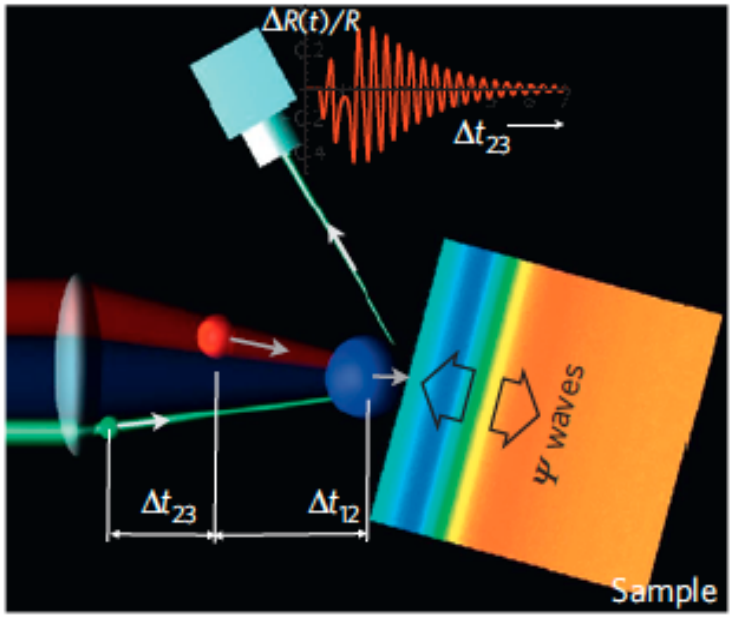
Matsunaga *et al*, PRL (2013)

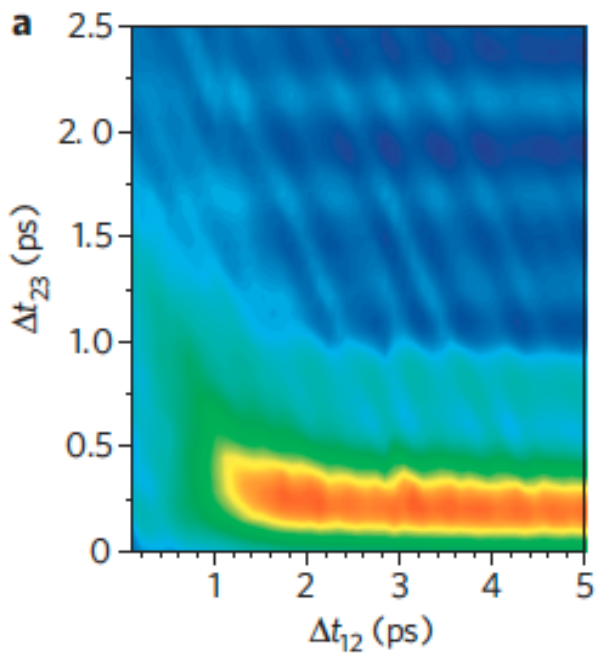
ARPES Pump Probe studies of Higgs mode in $TbTe_3$

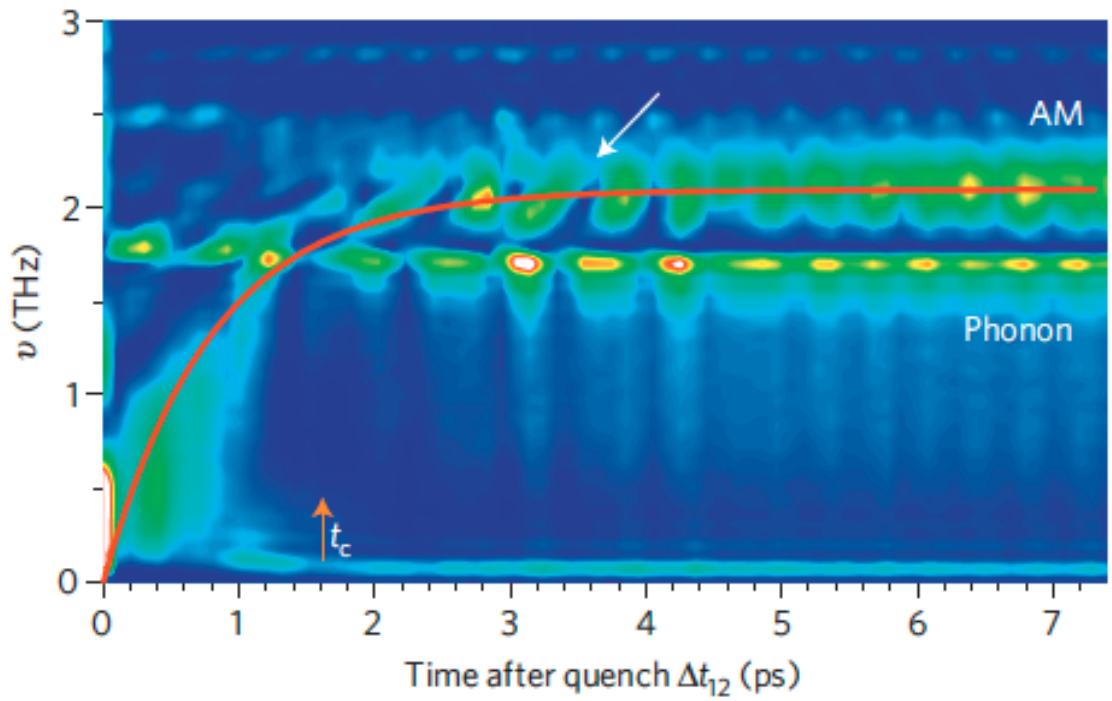


Schmitt *et al.*, Science (2008)

CDW Order Parameter Dynamics in $TbTe_3$ ripples due to `Higgs' mode (AM)

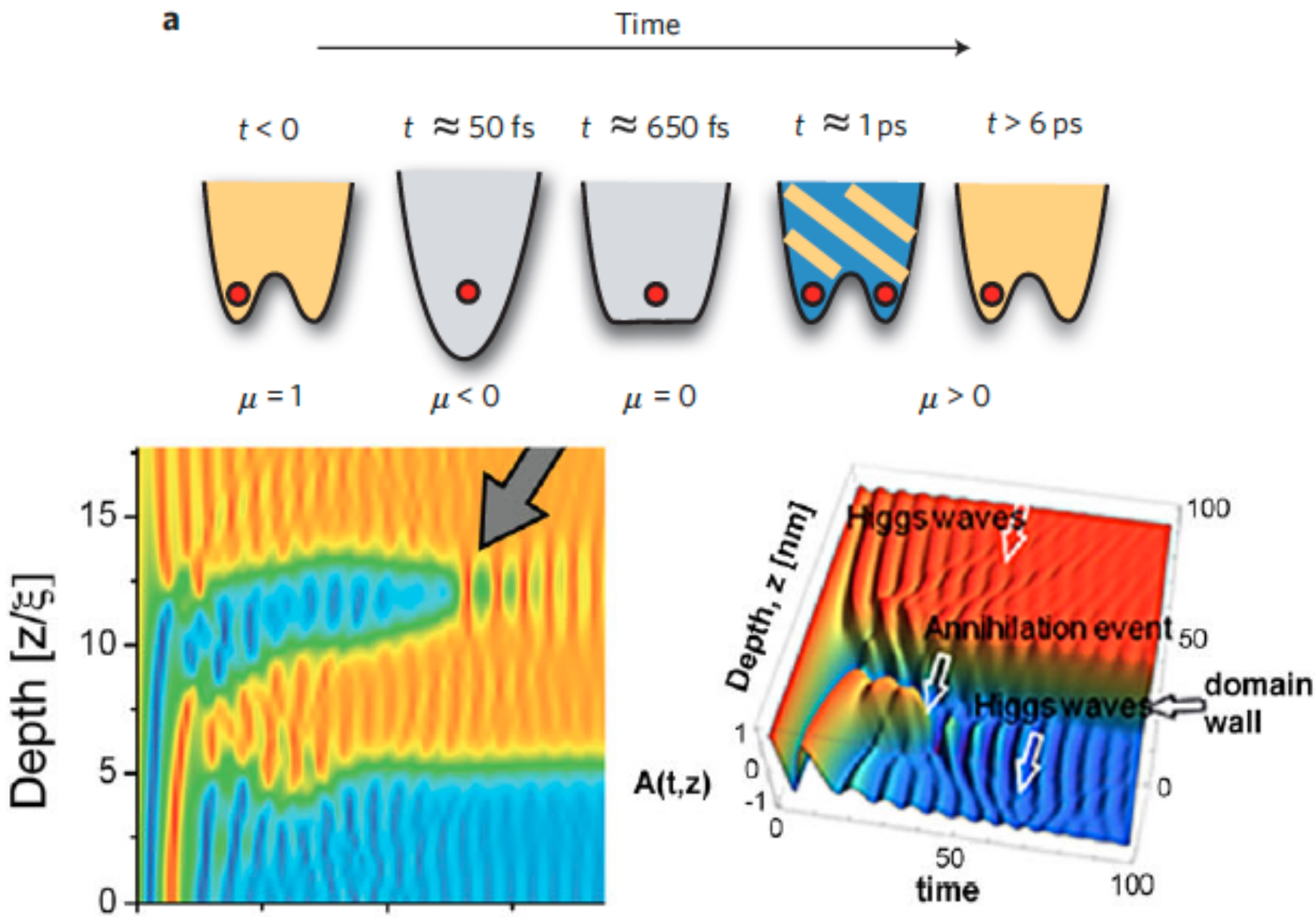






Yusupov *et al.*, Nat. Phys. (2010)

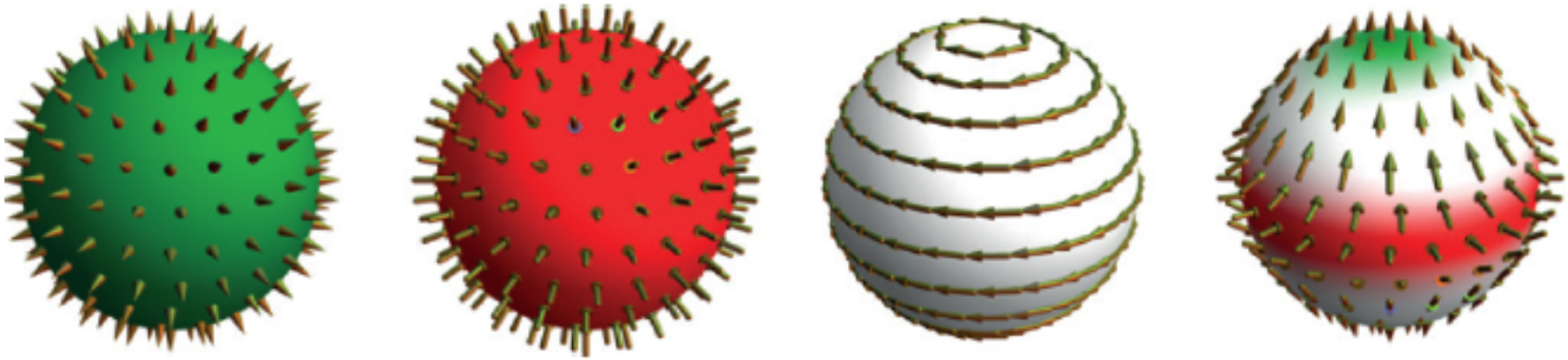
CDW order parameter domain creation and annihilation in $TbTe_3$ (Kibble-Zurek mechanism)



Yusupov *et al.*, Nat. Phys. (2010)

Multipole expansion of magnetic field in a solid where
m is the magnetic dipole moment; a the magnetic monopole;
t the polar toroidal moment (anapole); q the magnetic quadrupole

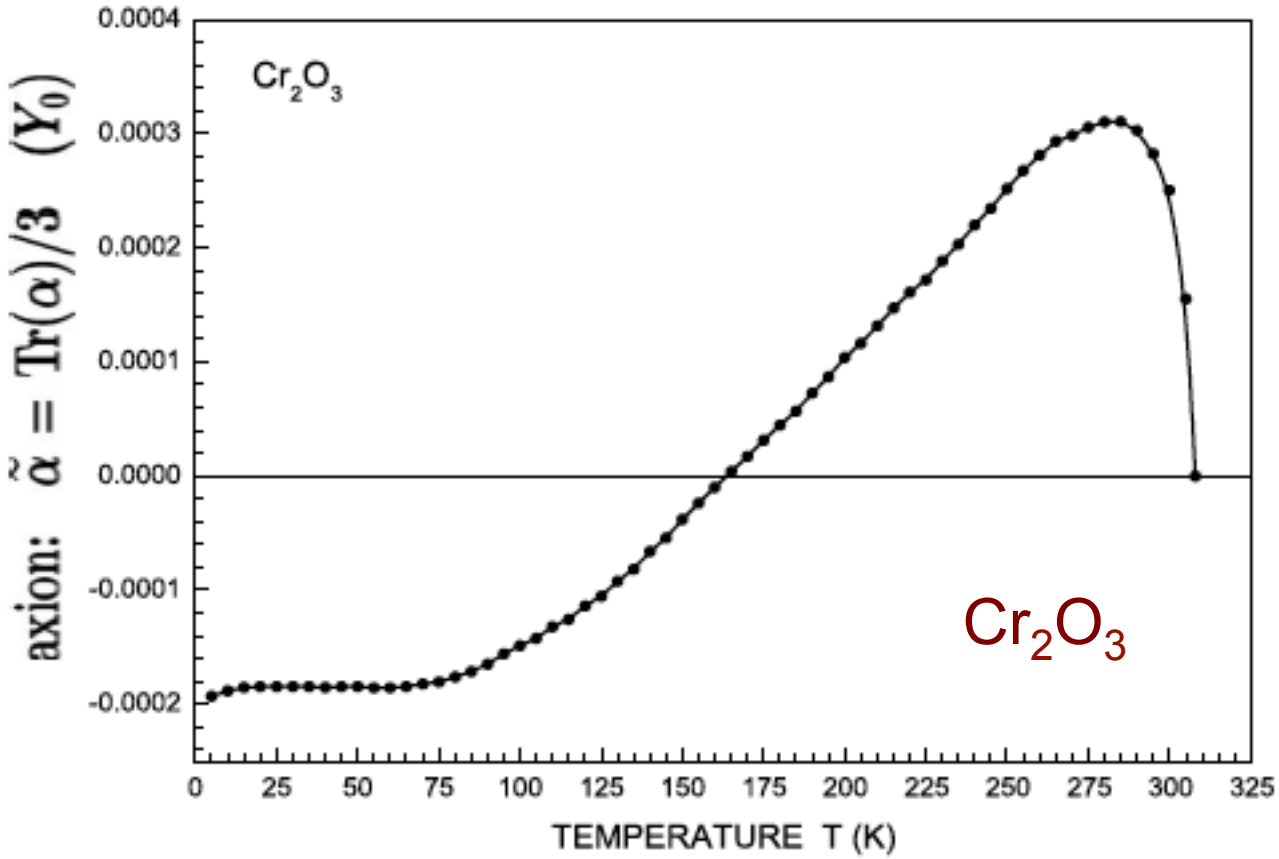
$$H_{\text{int}} = -\boldsymbol{m} \cdot \boldsymbol{H}(0) - a(\nabla \cdot \boldsymbol{H})_{r=0} - \boldsymbol{t} \cdot [\nabla \times \boldsymbol{H}]_{r=0}$$
$$-q_{ij}(\partial_i H_j + \partial_j H_i)_{r=0} - \ldots .$$



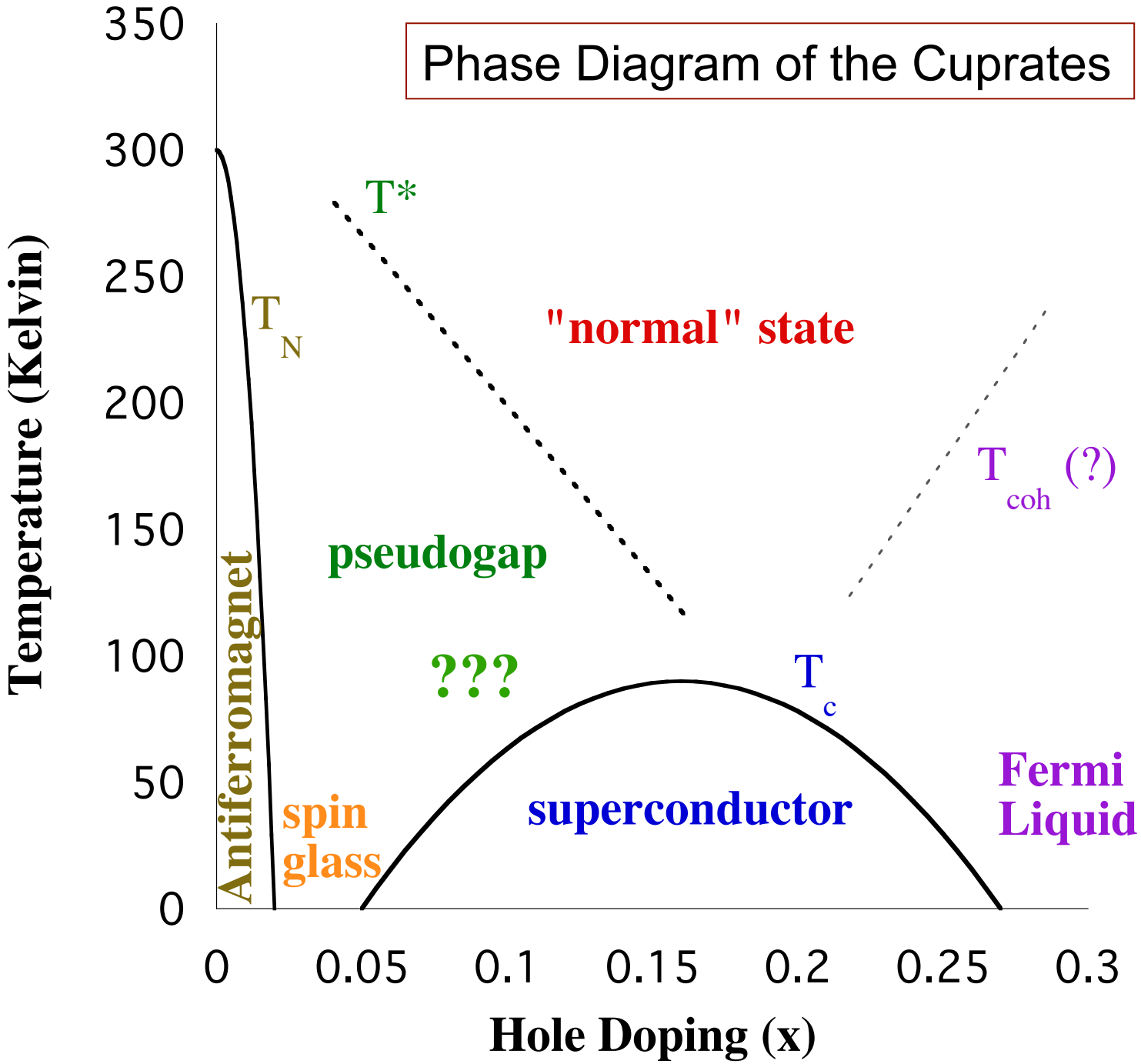
+ monopole - monopole z anapole z^2 quadrupole

Dubrovik & Tugushev, Phys Rep (2000)
Di Matteo, J Phys D (2012)
Spaldin *et al*, PRB (2013)

The “a” term leads to axion electrodynamics
In a magneto-electric material, this is defined as the trace of α, with $H_{int} = \alpha_{ij} E_i H_j$

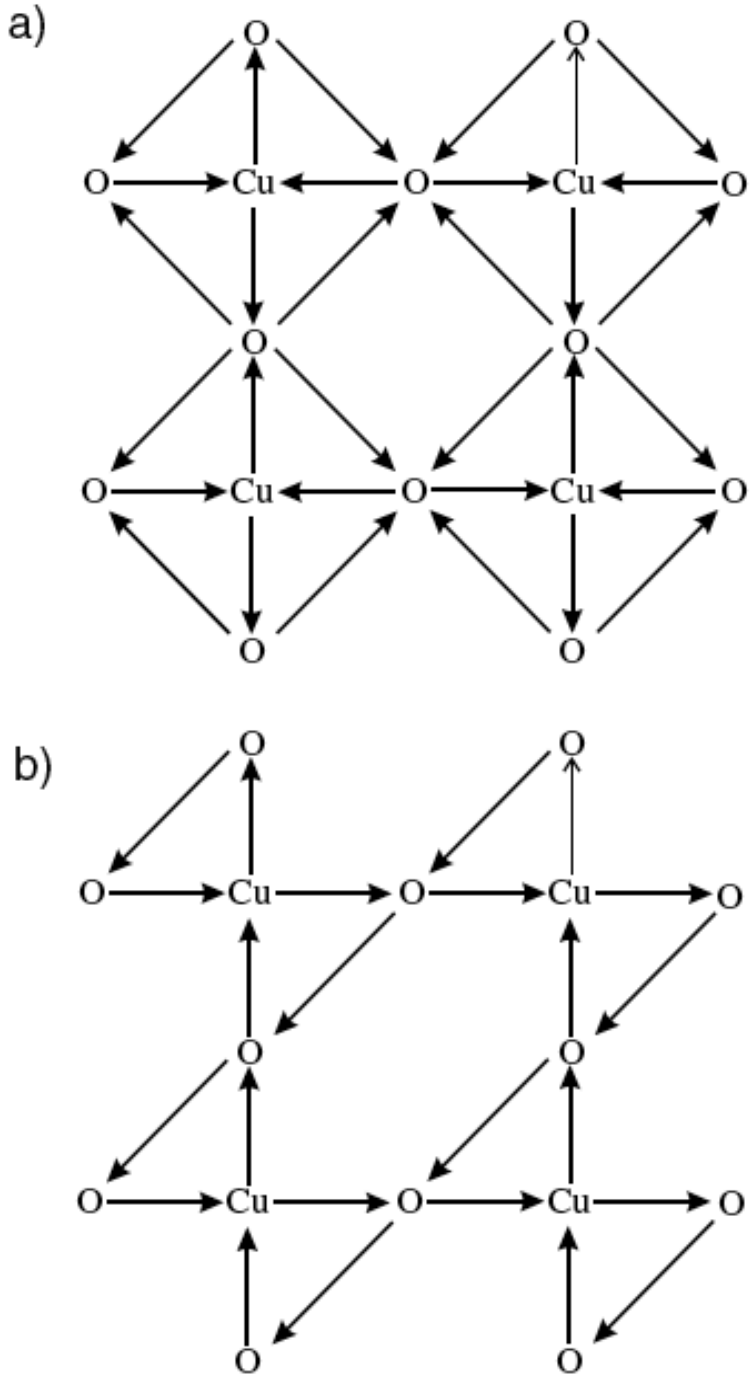


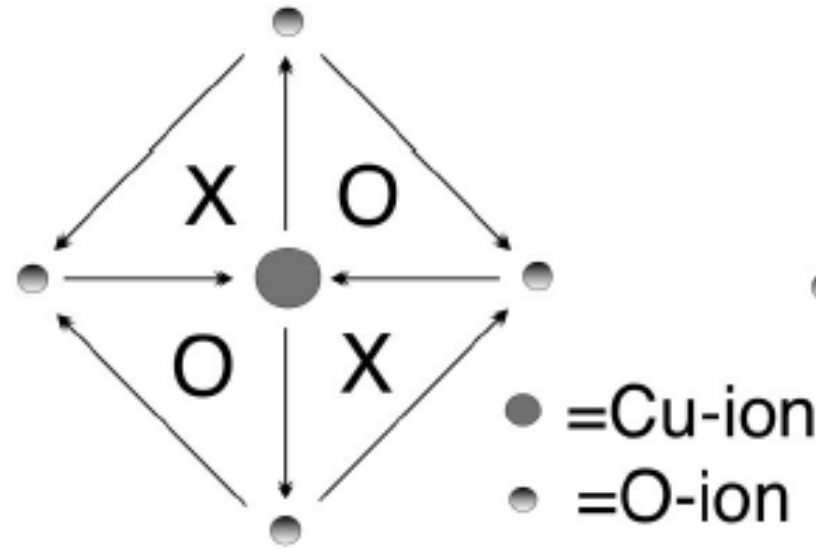
Hehl *et al*, PRB (2008)

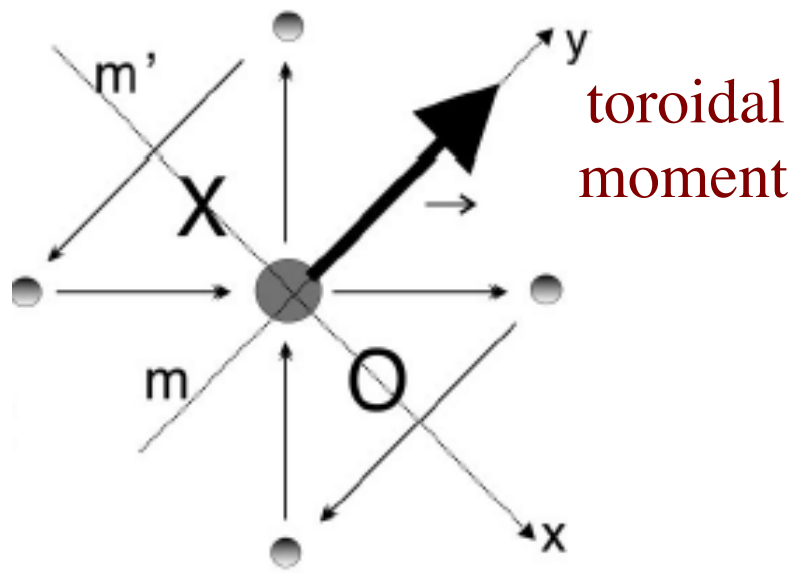


Norman, Pines, Kallin, Adv Phys (2005)

Orbital Currents in Cuprates

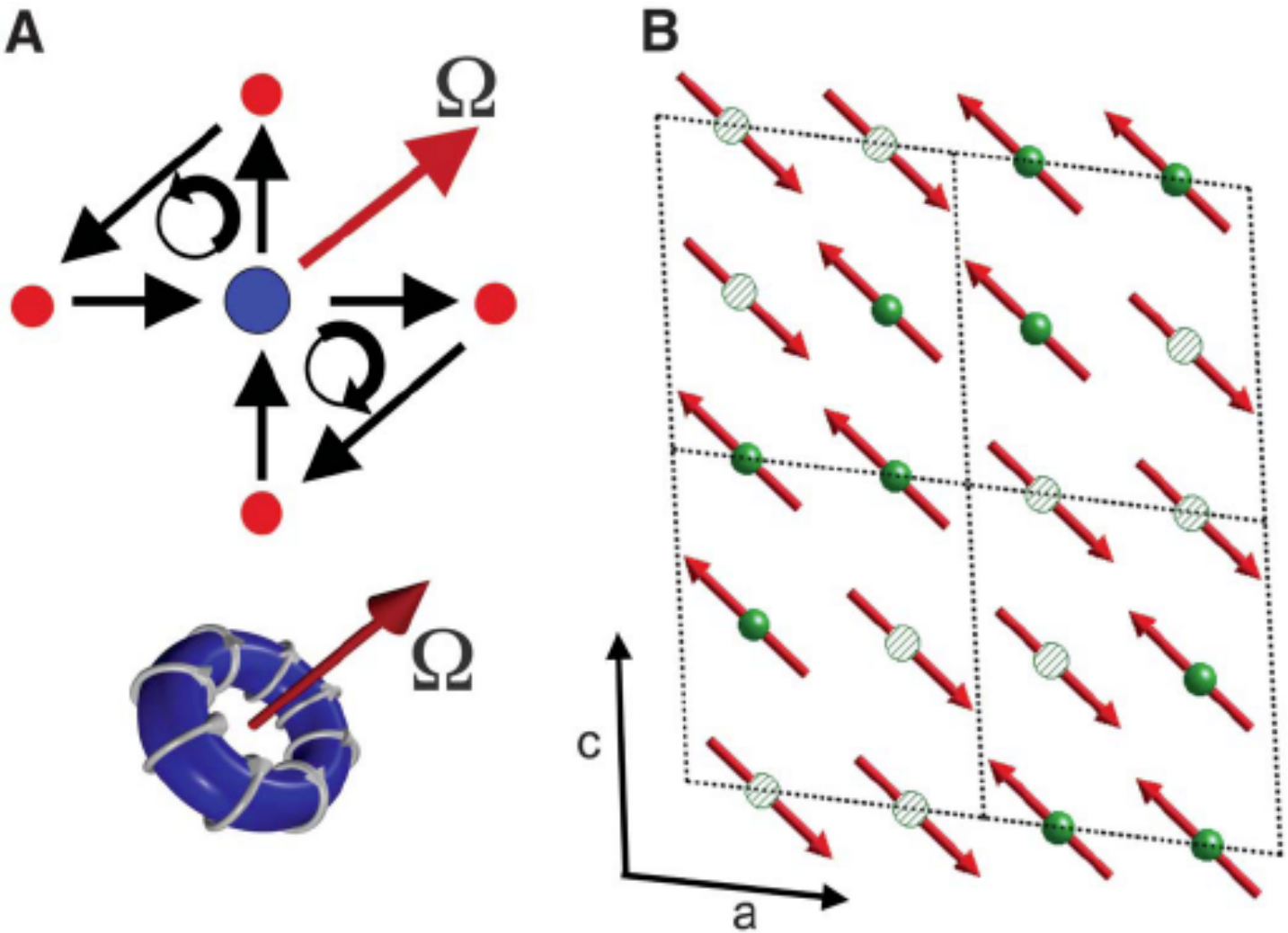






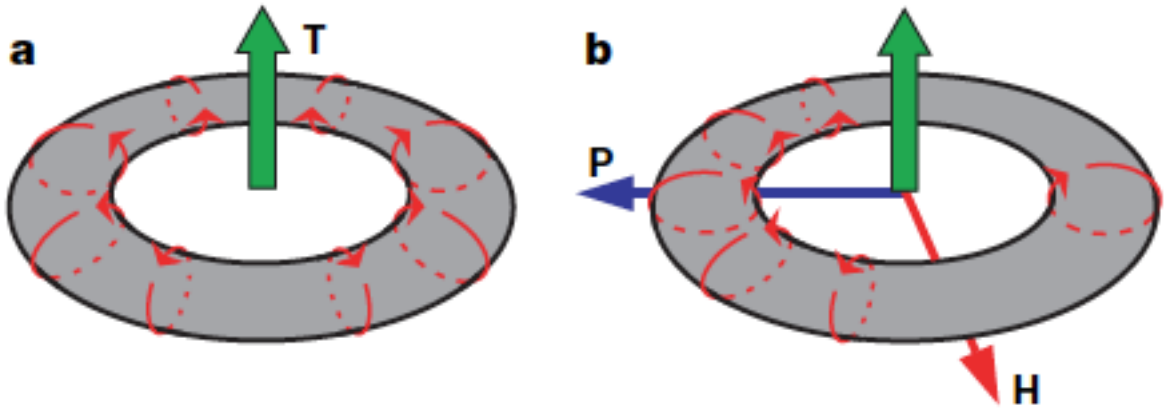
Simon & Varma, PRL (2002)
Di Matteo & Varma, PRB (2003)

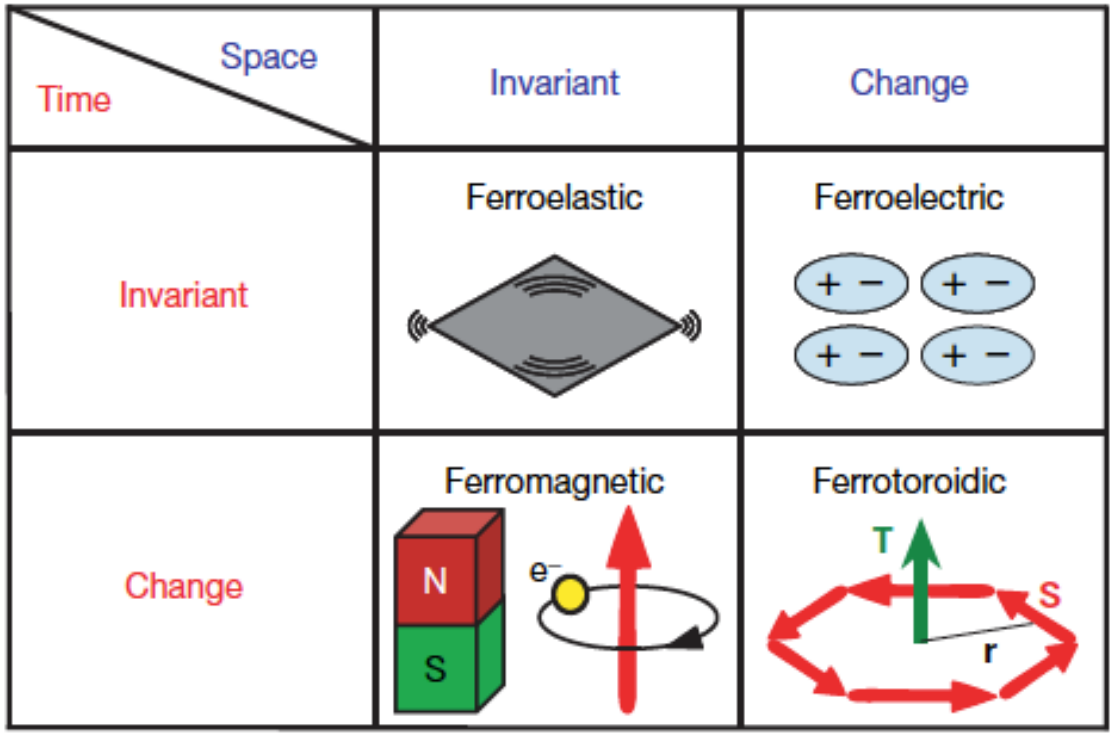
Toroidal Moments in the Commensurate AF Phase of CuO?

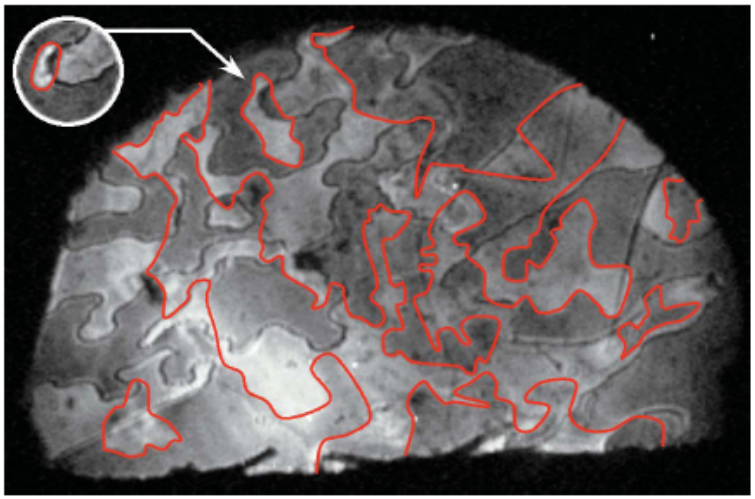


Scagnoli *et al*., Science (2011)

Toroidal moments have been observed in $LiCoPO_4$ (used as a cathode in Li-ion batteries) by optical second harmonic generation

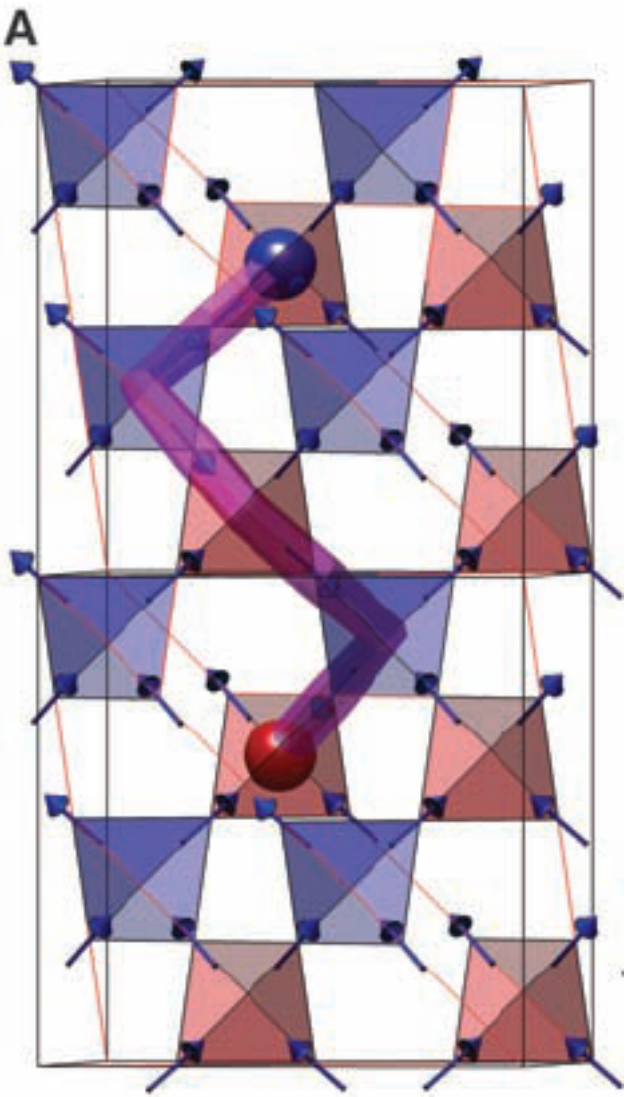






Van Aken *et al.*, Nature (2007)

Leonard: Are you really going to be on NPR?
Sheldon: Yes, they're interviewing me by phone from my office, regarding the recent so-called discovery of magnetic monopoles in spin-ices. It's pledge week and they're trying to goose the ratings with a little controversy.

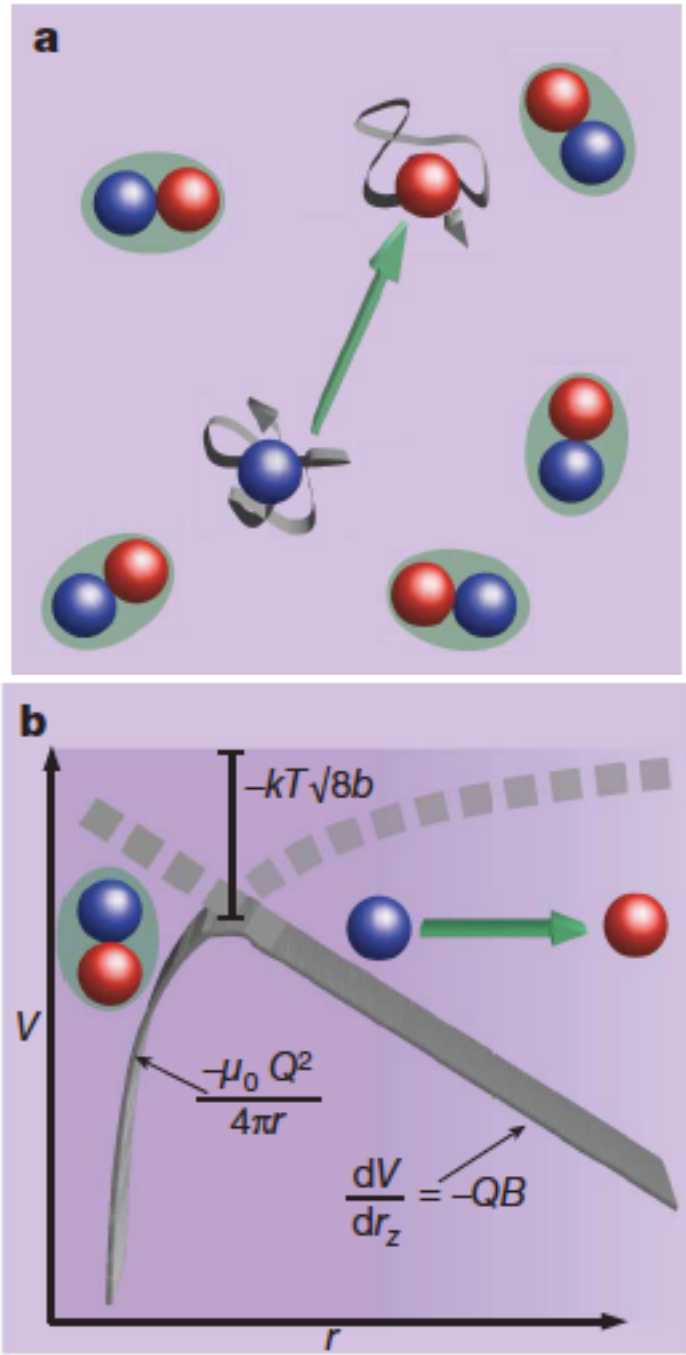


Pyrochlore magnets obey the spin ice rules (two spins pointing in, two pointing out, in each tetrahedron).

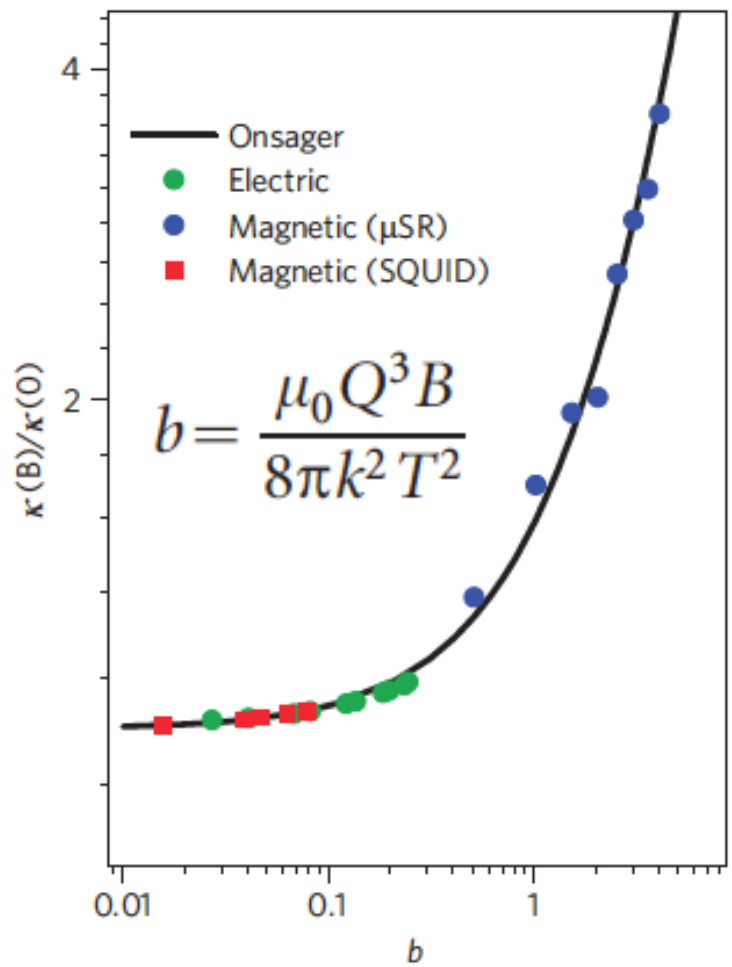
Flipping a spin creates a monopole-antimonopole pair, which as they move apart, create a “Dirac string” of flipped spins in their wake.

Castelnovo *et al*, Nature (2008)
Bramwell *et al*, Nature (2009)
Morris *et al*, Science (2009)
Fennell *et al*, Science (2009)

Spin ice conducts magnetic charge like an electrolyte ("magnetolyte")

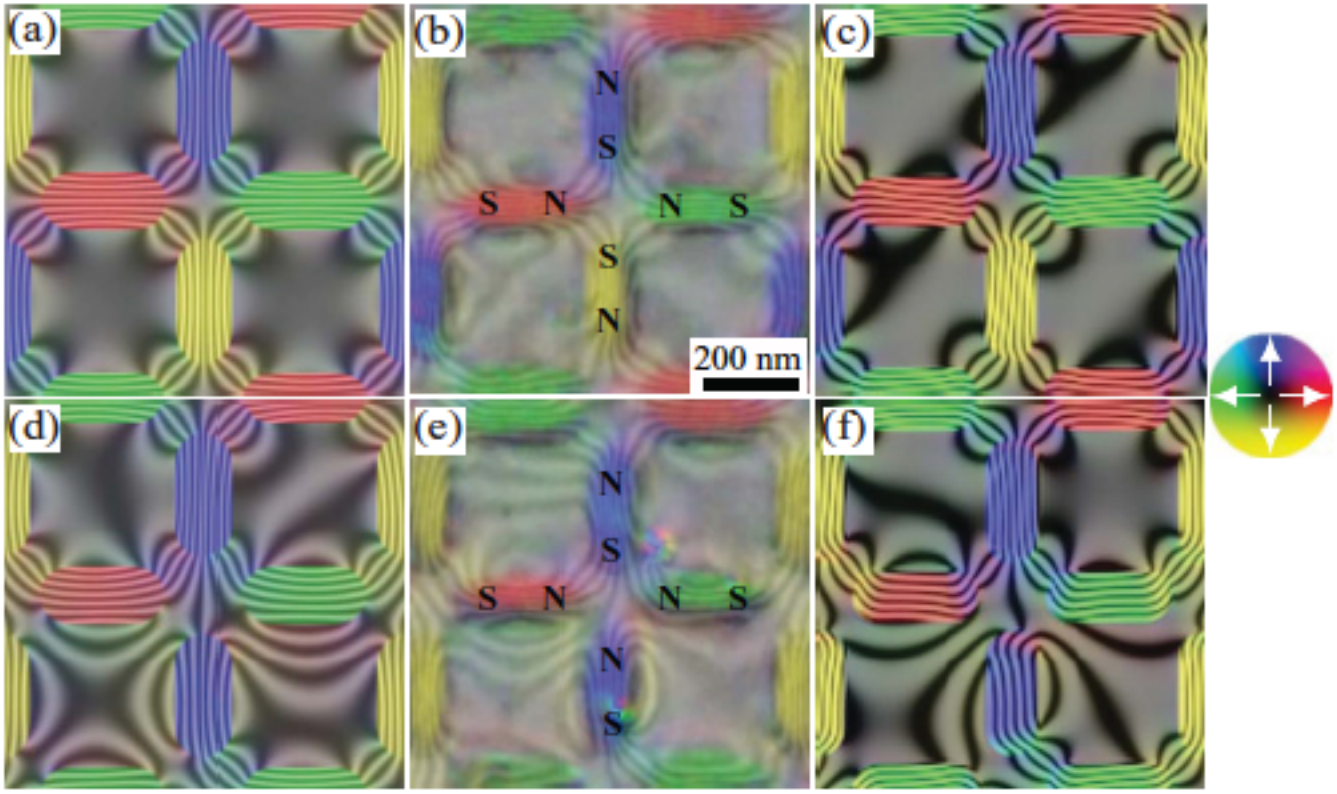


$$\frac{n_f^{eq}(B)}{n_f^{eq}(0)} = \frac{\kappa(B)}{\kappa(0)} = 1 + \frac{b}{2} + \frac{b^2}{24} + \cdots$$



Bramwell *et al*, Nature (2009)
Giblin *et al*, Nat Phys (2011)

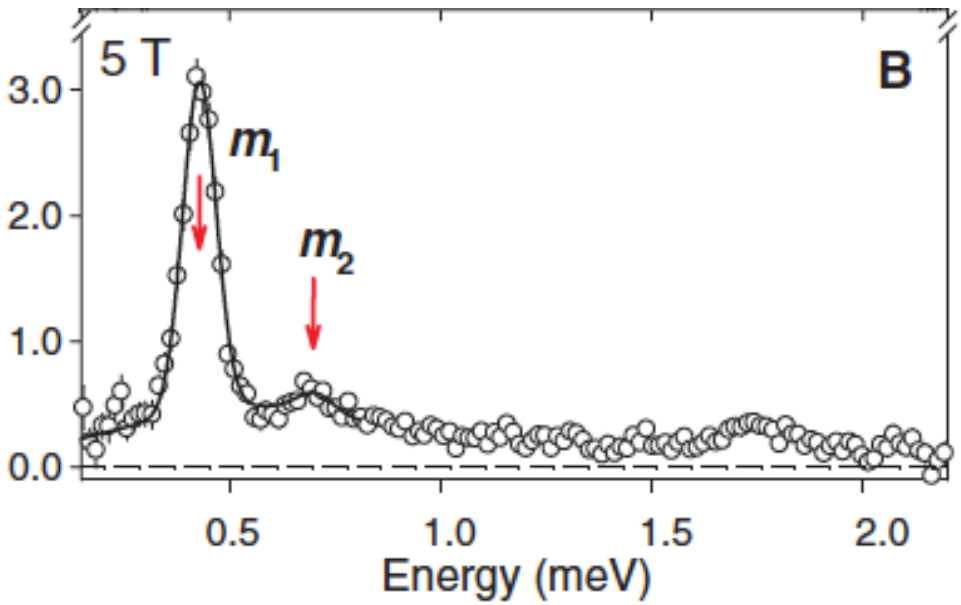
Magnetic Monopole Defects in Artificial Spin Ice (imaged with Lorentz TEM)

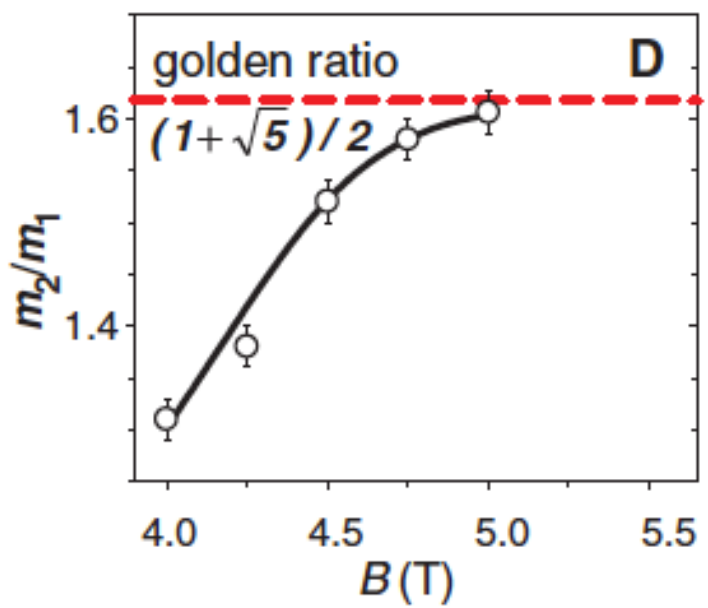


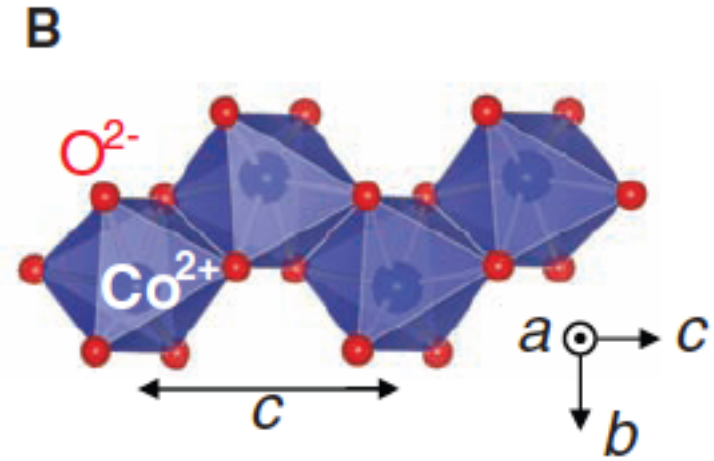
Phatak *et al*, PRB (2011)

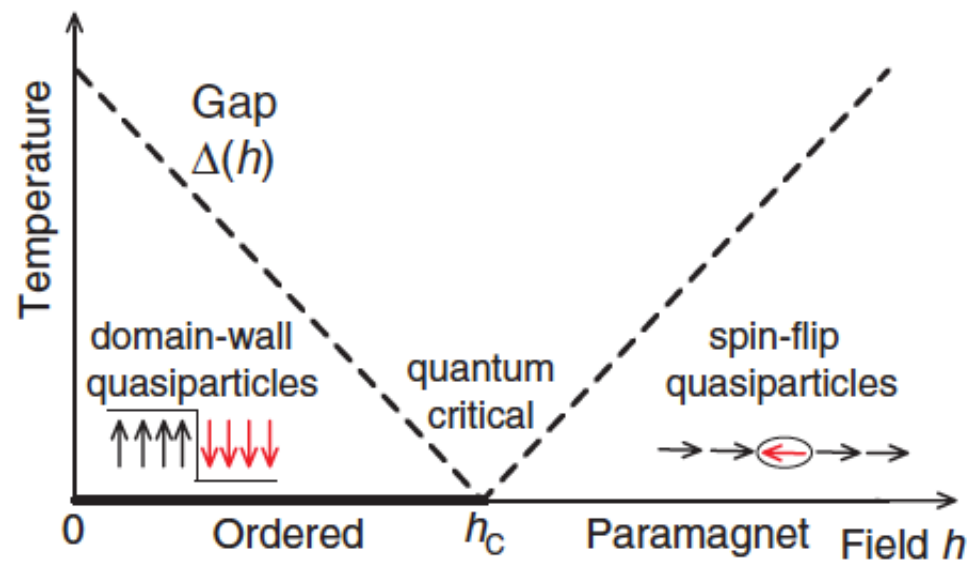
Emergent E_8 Symmetry in an Ising Spin Chain ($CoNb_2O_6$)

$$H = \Sigma_i - JS_i^z S_{i+1}^z - hS_i^x$$



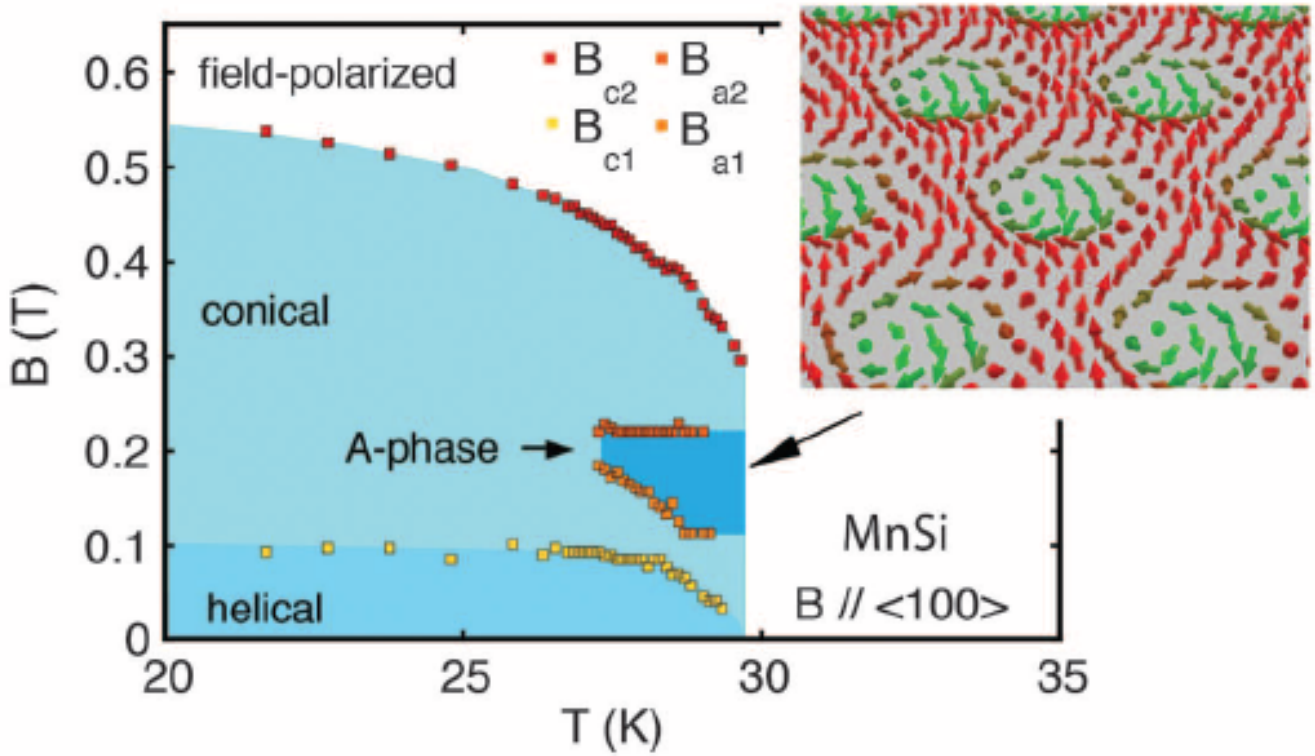




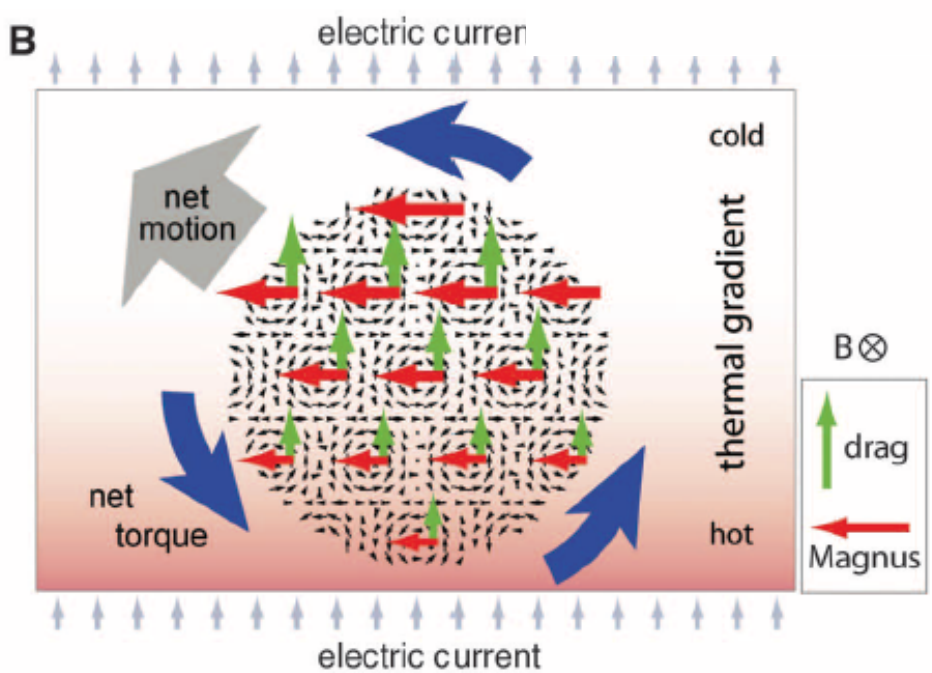


Zamolodchikov, IJ MPA (1989)
Coldea *et al*, Science (2010)

Skyrmion Crystal in the “A” phase of the helical magnet MnSi

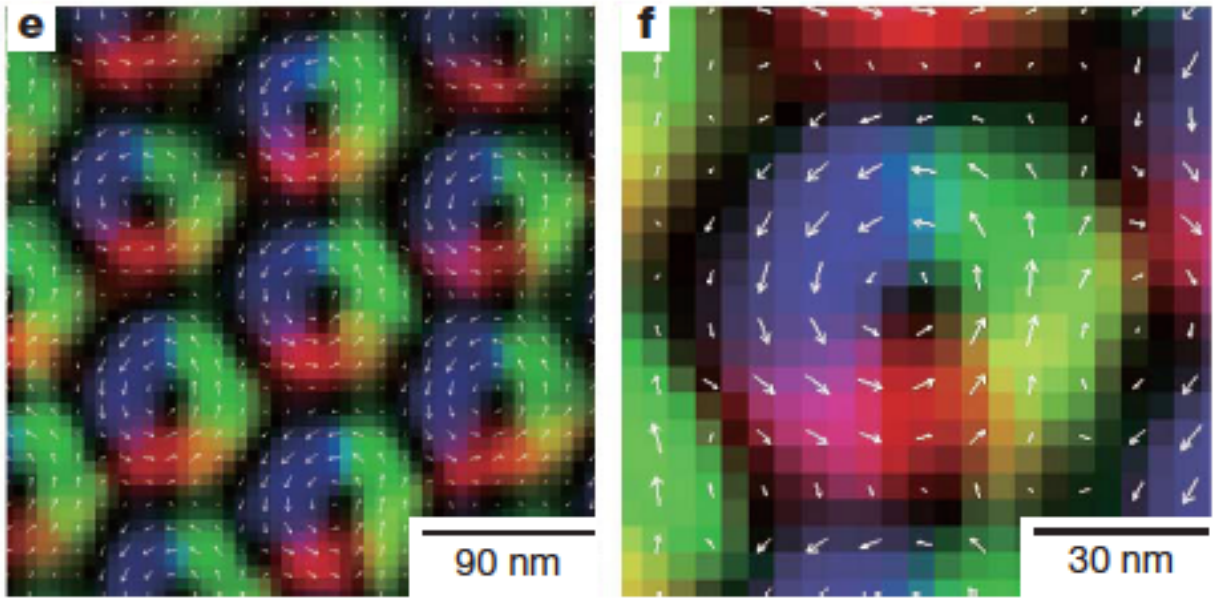


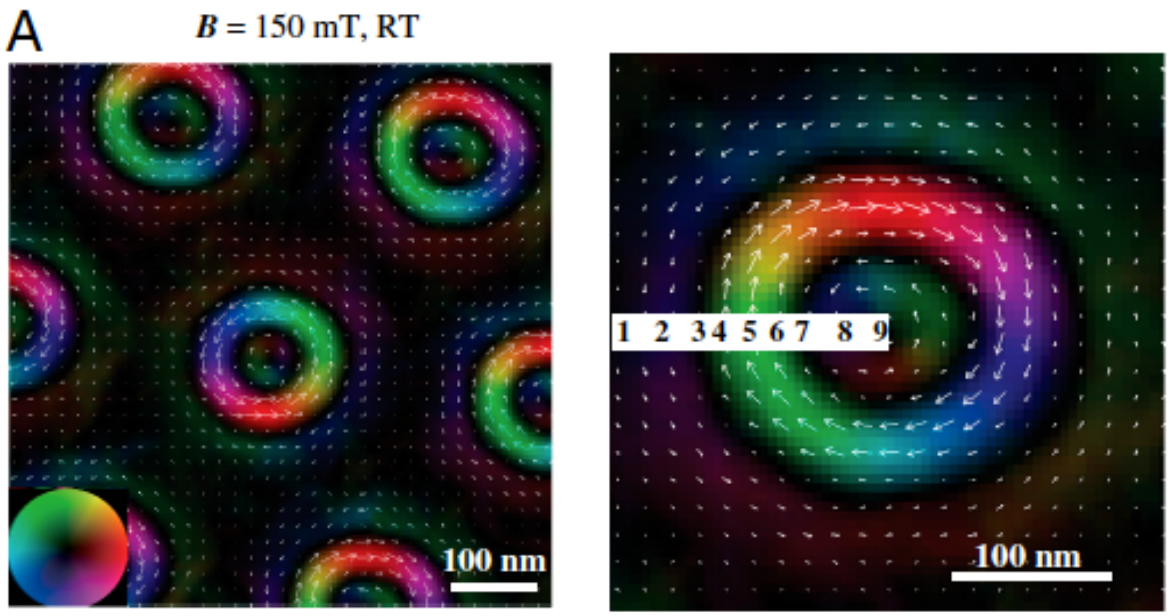
Spin torque



Muhlbauer *et al*, Science (2009)
Jonietz *et al*, Science (2010)

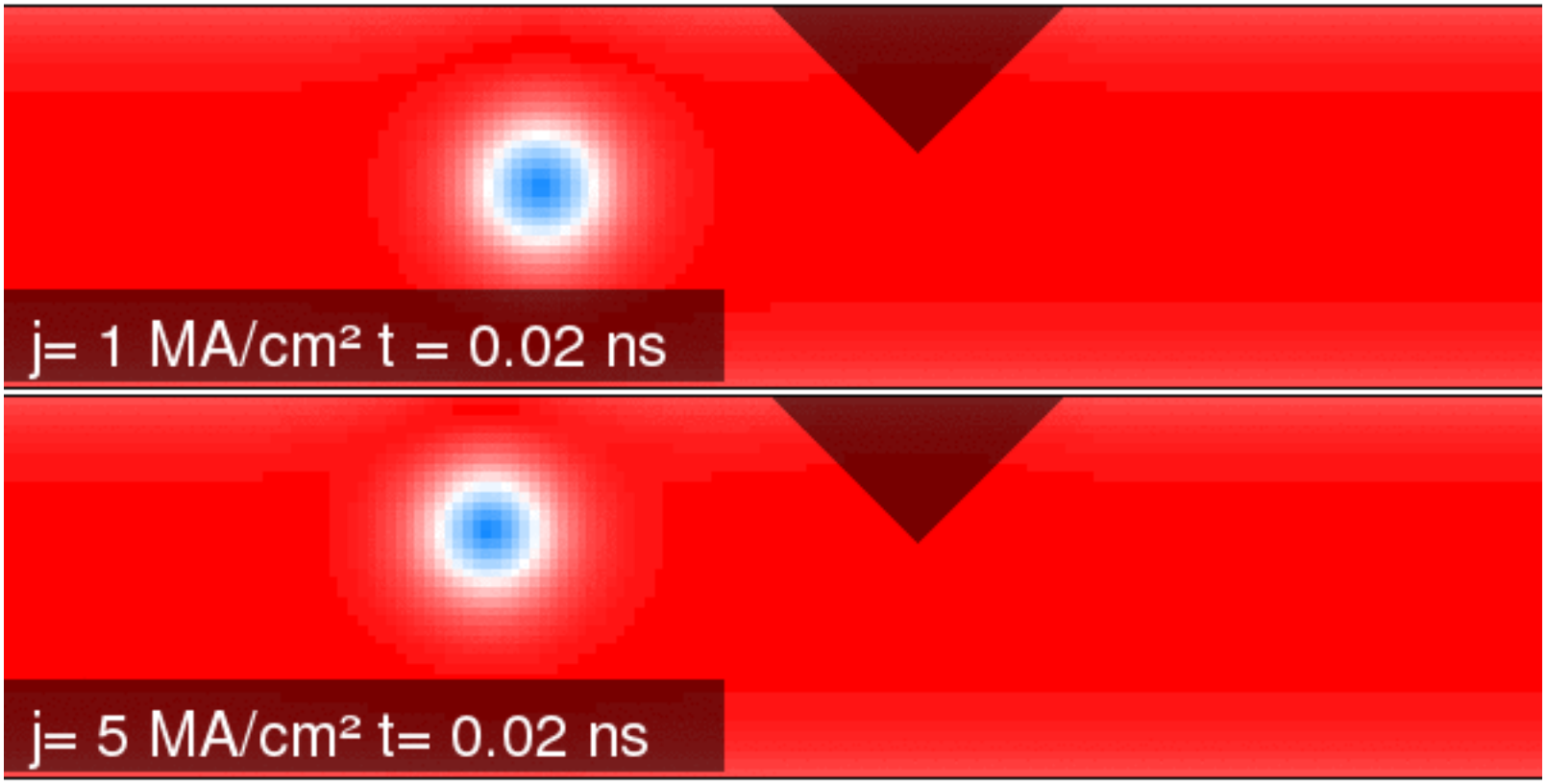
Skyrmions in Co doped FeSi (top) and Sc doped Ba ferrite (bottom) as imaged by Lorentz TEM





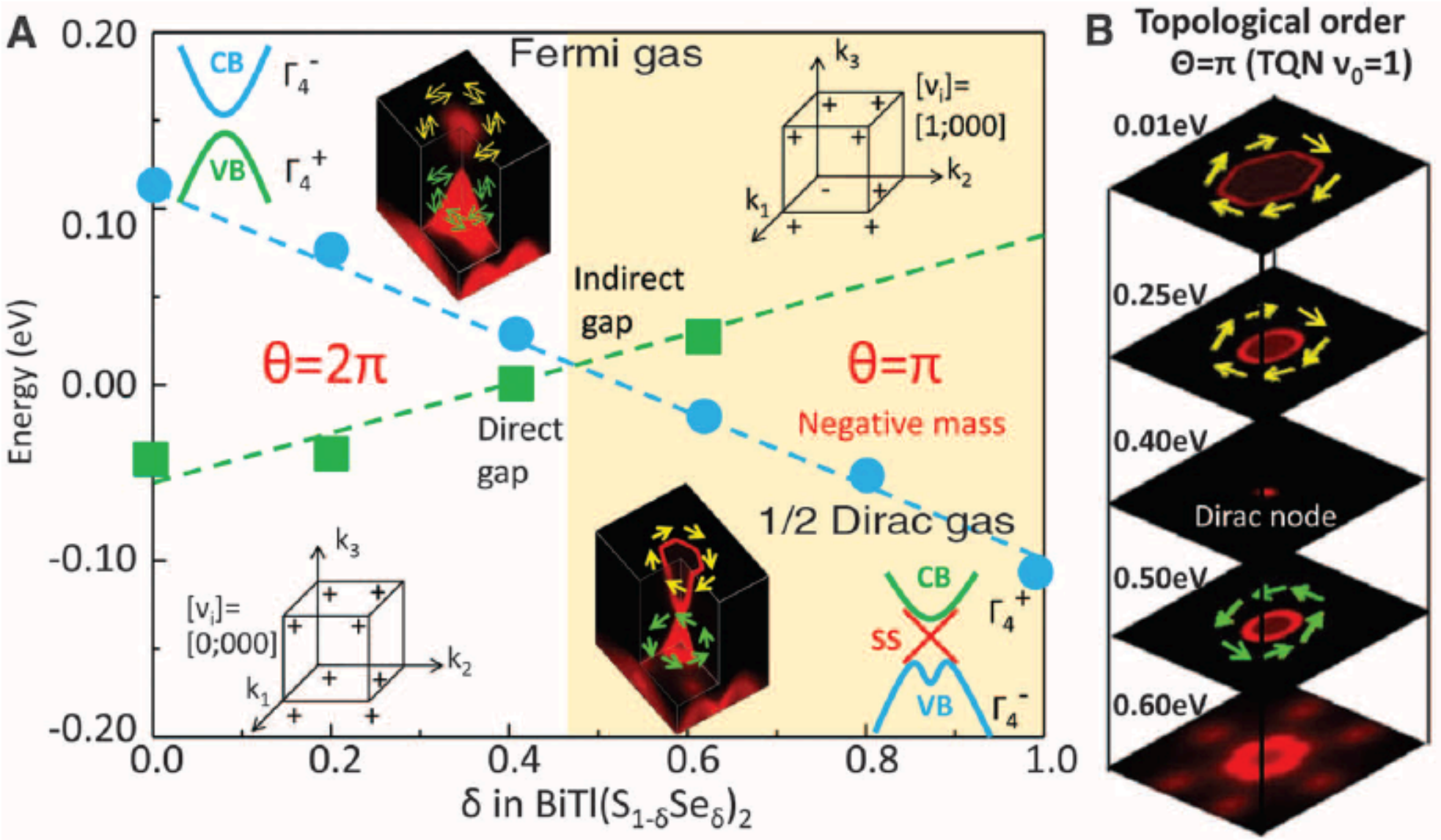
Yu *et al*, Nature (2010)
Yu *et al*, PNAS (2012)

Micromagnetic Simulations of Current Induced Skyrmion Motion (possibly leading to spintronic storage or logic devices)



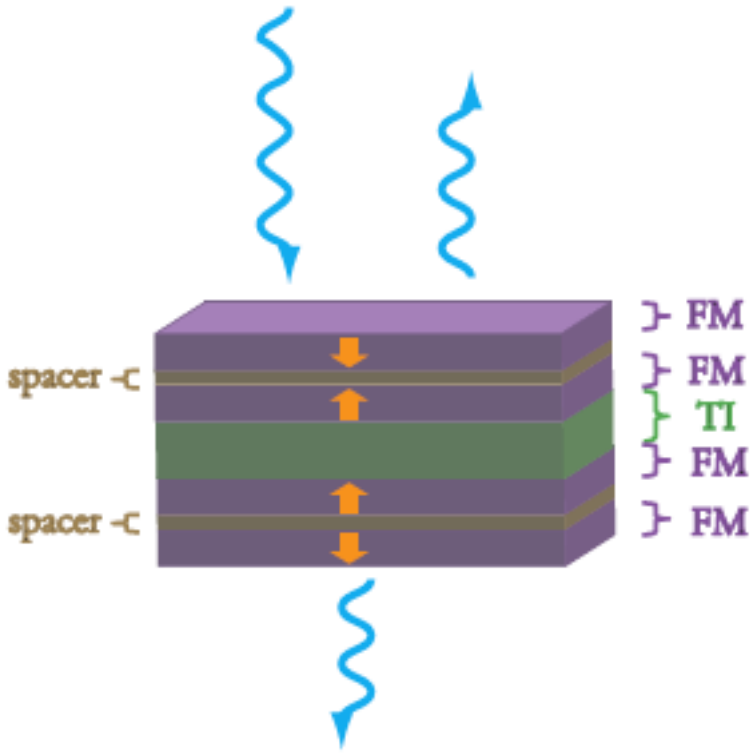
Fert *et al*, Nature Nano (2013)

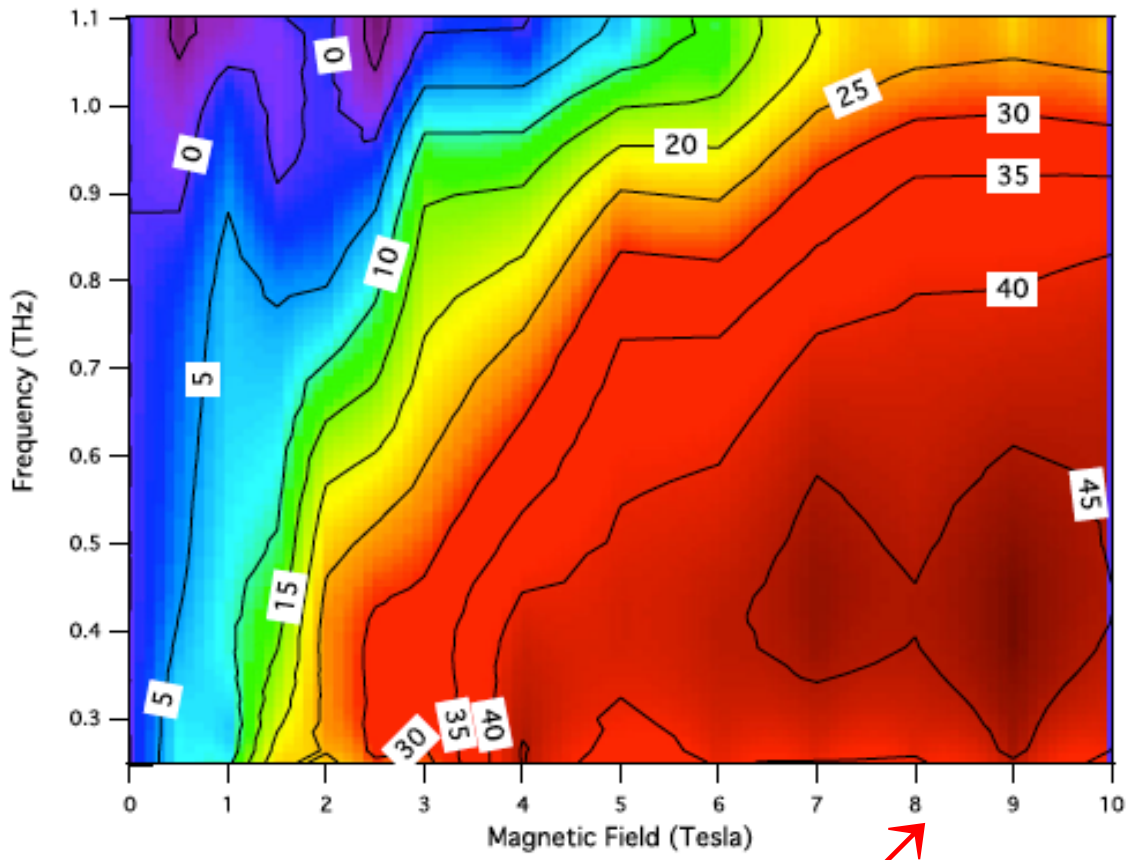
Topological Surface States in Spin-orbit Insulators - $BiTl(SSe)_2$



Xu *et al*, Science (2011)

Axion electrodynamics as measured by Kerr or Faraday effect

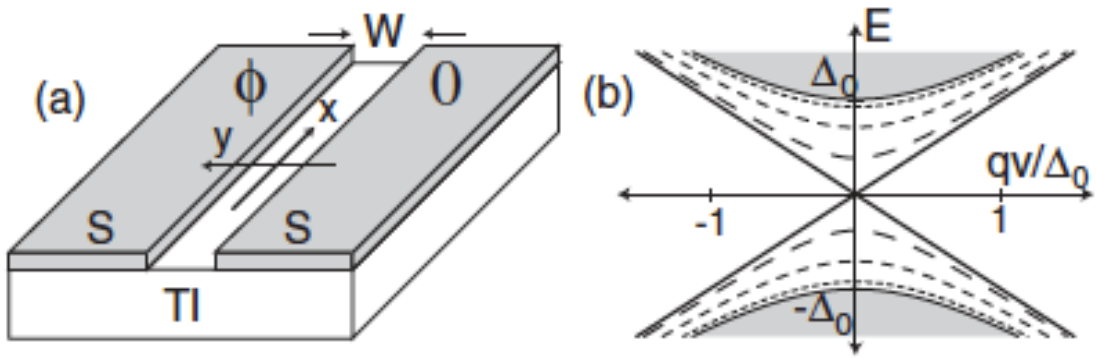


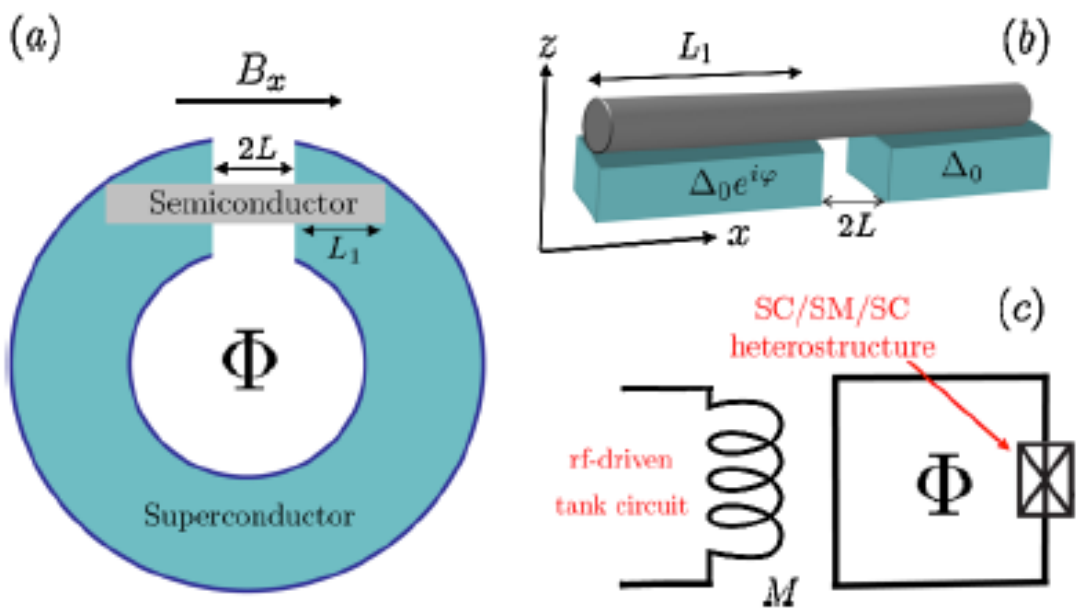


Kerr angle of 45°

Qi, Hughes, Zhang, PRB (2008)
Maciejko *et al*, PRL (2010)
Tse & MacDonald, PRL (2010)
Aguilar *et al*, PRL (2012)

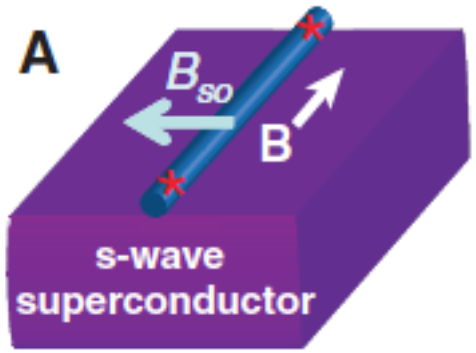
Junctions between a topological insulator (or spin-orbit wire) and a superconductor might localize a Majorana mode at the junction

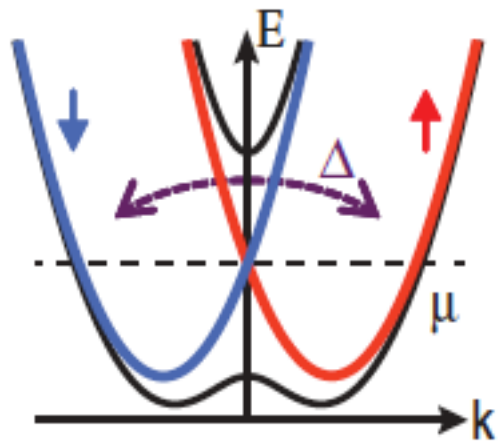




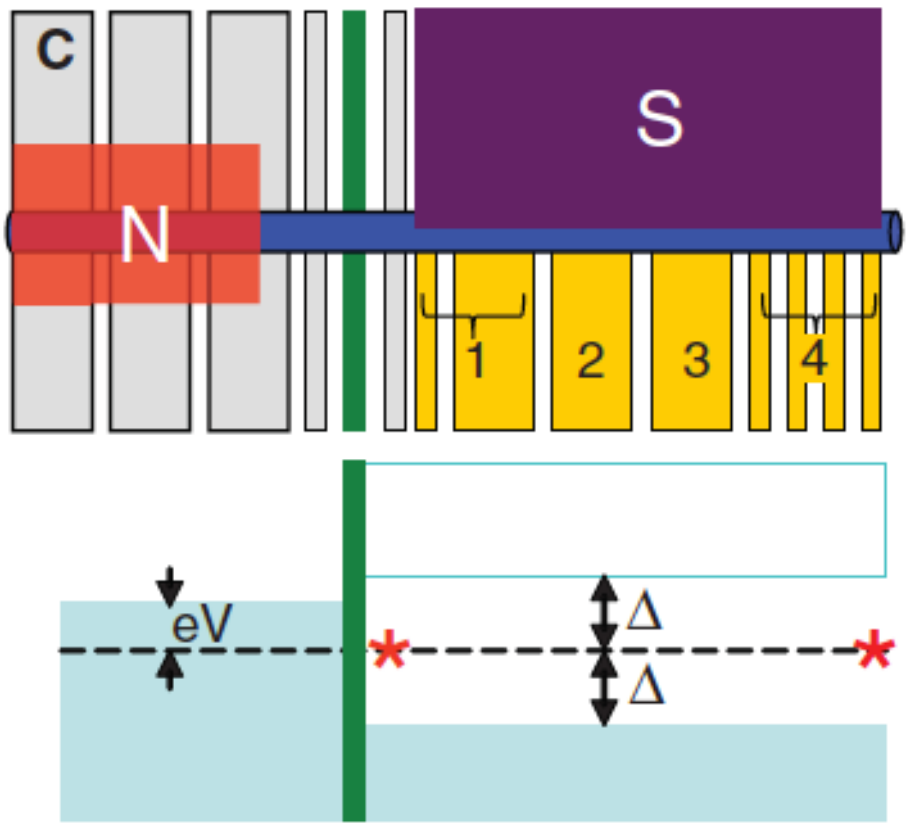
Kitaev, Phys Usp (2001)
Fu & Kane, PRL (2008)
Lutchyn *et al*, PRL (2010)
Oreg *et al*, PRL (2010)

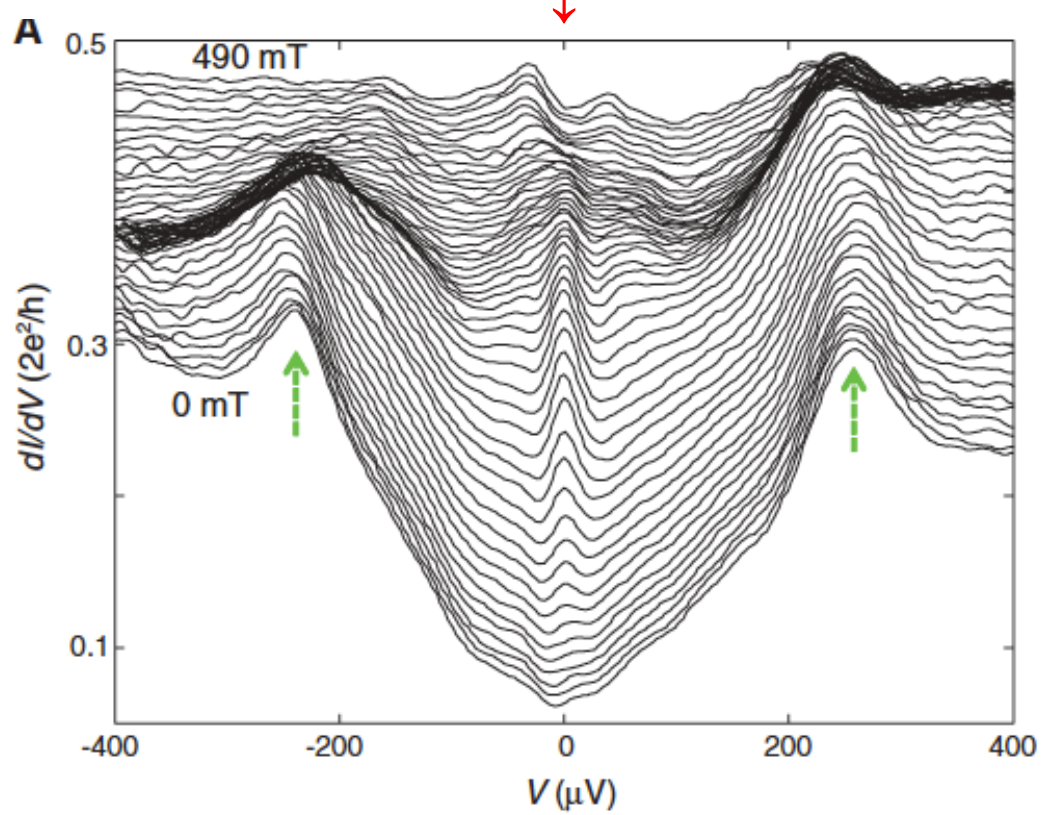
Majorana Fermions in an InSb nanowire on NbTiN ?





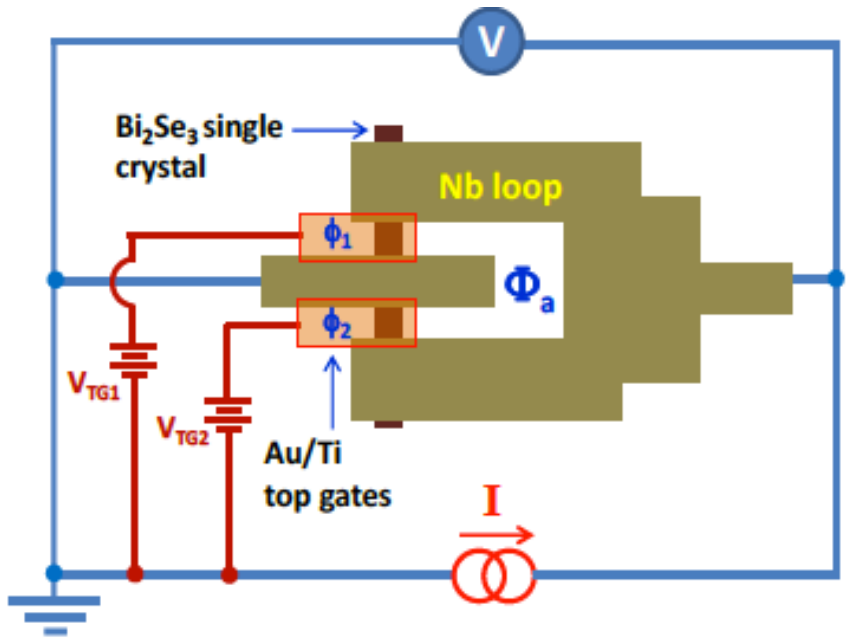
zero mode

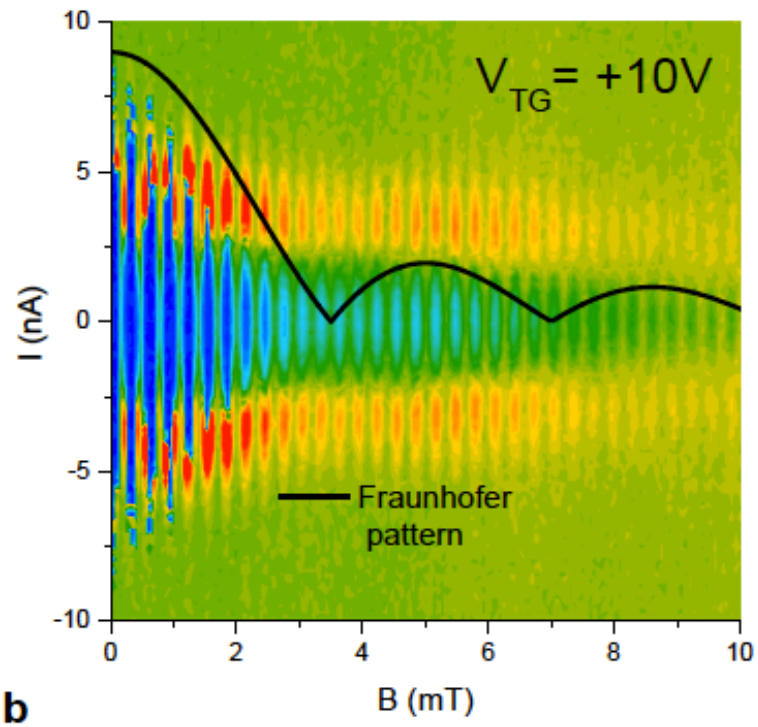


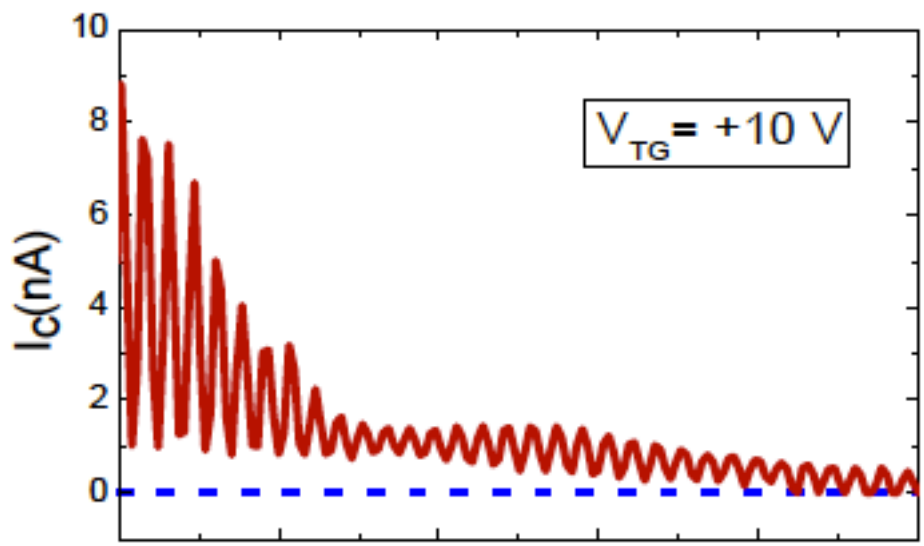


Mourik *et al*, Science (2012)

Majorana signature – 4π contribution to Josephson phase ?

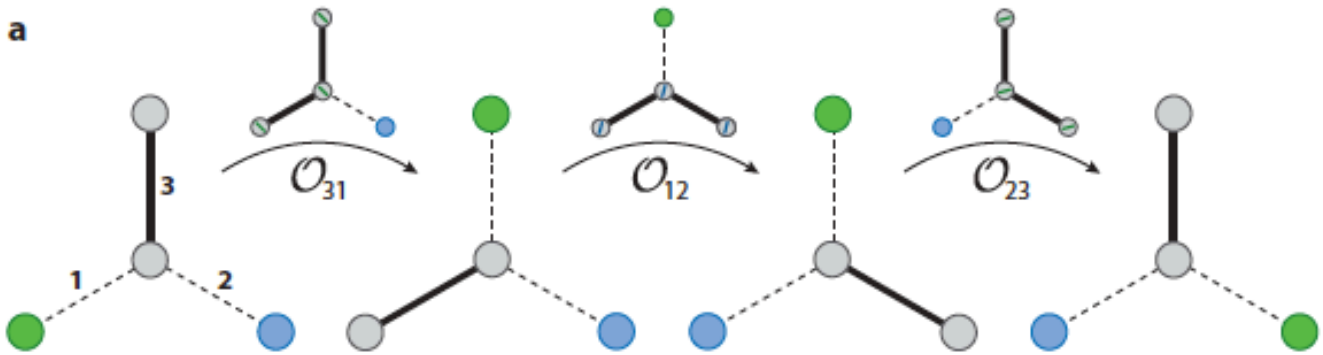


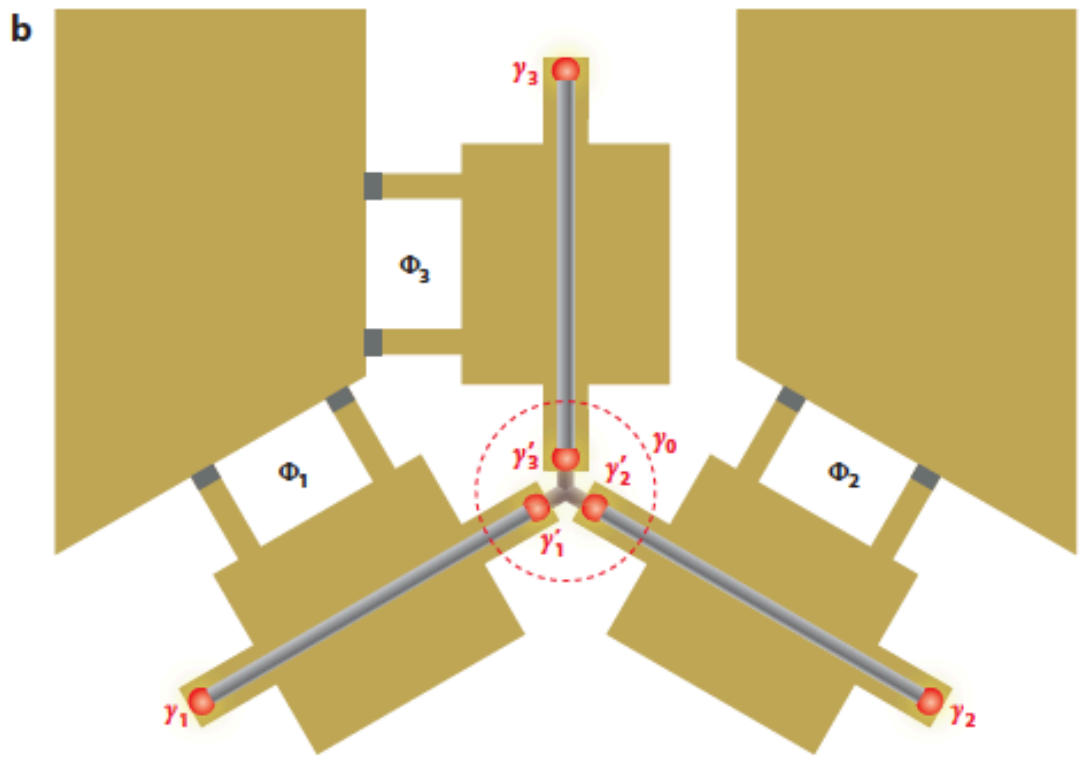




Kitaev, Phys Usp (2001)
Kurter *et al*, arXiv (2013)

Topological Quantum Computing by Braiding Majorana Fermions





$$\Psi \mapsto \exp\left(i\frac{\pi}{4}\sigma_z\right)\Psi$$

Kitaev, Phys Usp (2001)
Nayak *et al*, RMP (2008)
Alicea, RPP (2012)
Beenakker, ARCMP (2013)