Interaction-induced threshold singularities in tunneling via localized levels

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The current-voltage characteristic of the tunnel junction with a localized impurity level in the dielectric layer is studied. We take into account the fact that the electron tunneling from the level to the lead is accompanied by the formation of a positively charged hole bound to the impurity. The Coulomb interaction of electrons with the hole leads to power-law singularities in the transition rate, analogous to those arising in the problem of x-ray-absorption edges in metals. They manifest themselves in I-V characteristic as $I \sim (V - V_{\rm th})^{-\alpha}\theta(V - V_{\rm th})$, the exponent α being small when the spacing between impurity and junction leads is greater than the Fermi wavelength λ_F. Predicted dependences are most likely to be observed in junctions with leads made of a heavily doped semiconductor or in GaAs heterostructures, where the spacing may be of the order of λ_F.

I. INTRODUCTION

Recently the fabrication of very small tunnel junctions has become possible. The physics of electronic transport in these devices has attracted a lot of interest, both theoretical and experimental.[1] One of the peculiarities of small tunnel junctions is that a single impurity situated within the tunnel barrier may increase the conductance dramatically. The influence of the impurity is most significant if it forms a localized level with the energy E_i close to the Fermi level E_F in the junction leads. As it was shown in the works by Chaplik and Entin[2] and Lifshits and Kirpichenkov[3] the presence of a localized level in the tunnel barrier leads to a resonant rise in tunneling rate:

$$w_{kp} = \frac{2\pi}{\hbar}\frac{|T_k|^2|T_p|^2}{(E_i - \epsilon_k)^2 + (\Gamma_L + \Gamma_R)^2}\delta(\epsilon_k - \epsilon_p), \qquad (1)$$

where $T_{k(p)}$ is the amplitude of tunneling from state k (p) in the left (right) lead of the junction to the localized level, ϵ_k and ϵ_p are corresponding energies, and Γ_L, Γ_R are the widths of the localized level due to the tunneling into the left and right leads correspondingly (see, e.g., Ref. 4). The resonance (1) shows up in the tunneling current which may be determined as

$$I = e\sum_{kp}(n_k - n_p)w_{kp}, \qquad (2)$$

where n_k and n_p are the Fermi occupation numbers of the states in the left and right leads of the junction. In particular, at small bias $V = (E_F^L - E_F^R)/e$ the current is linear in V, and the zero temperature conductance associated with the impurity has the form

$$G = \frac{4e^2}{\pi\hbar}\frac{\Gamma_L\Gamma_R}{(E_i - E_F)^2 + (\Gamma_L + \Gamma_R)^2}. \qquad (3)$$

Resonant peaks in the dependence of conductance versus E_F (gate voltage) have been observed in silicon metal-oxide-semiconductor field-effect transistors (MOSFET's).[5,6]

Another possible way to observe the tunneling via resonant level is by measuring the current-voltage characteristic.[7] In the following we consider the I-V characteristic in a particular case of tunnel junction with single impurity situated somewhat closer to the left lead, $\Gamma_L \gg \Gamma_R$, and E_i below the Fermi energy: $E_F - E_i \gg \Gamma_L$. Conventional theory based on Eqs. (1) and (2) predicts the steplike dependence

$$I(V) = \frac{4e^2}{\hbar}\Gamma_R\theta(V - V_{\rm th}), \qquad (4)$$

(dashed line in Fig. 1). Here $\theta(x)$ is the unit step function; threshold voltage $V_{\rm th}$ may be found from the condition $E_i = E_F^R(V_{\rm th})$. Within this approach one also finds that in fact the step function (4) has finite width $\delta V \sim \Gamma_L/e$; we may neglect it as long as the voltage V is not very close to the threshold value: $|V - V_{\rm th}| \gg \Gamma_L/e$. Apparently the current-voltage characteristic may be also measured in two-dimensional (2D) junctions.[5,6] In this case the threshold voltage $V_{\rm th}$ determined by the energy of the localized level E_i may be adjusted by changing the gate voltage.

The expression (4) may be also understood as follows. The condition $\Gamma_L \gg \Gamma_R$ means that the localized level occupation number is determined by the Fermi energy in the left lead; moreover, it equals 1 because $E_i < E_F < E_F^L$. (Throughout this paper the temperature is supposed to be zero.) So the current in the junction is determined by the small rate Γ_R of tunneling between the impurity and the right lead. The step function $\theta(V - V_{\rm th})$ reflects the fact that at $V < V_{\rm th}$ the energy of the localized state is lower than the Fermi level in the right lead and the

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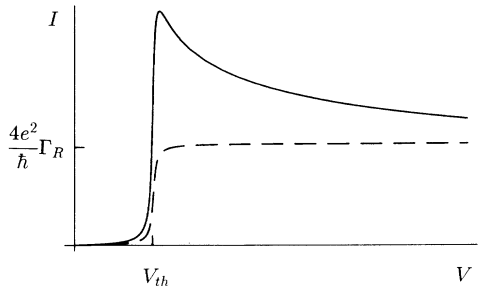

FIG. 1. Schematic view of the I-V characteristic of the tunnel junction with a single impurity. The case of noninteracting electrons (dashed line) is described by the steplike function (4). The solid line corresponds to nonvanishing Coulomb interaction, Eq. (8). The threshold singularities are smeared out at $|V - V_{\rm th}| \lesssim \Gamma_L/e$.

tunneling is forbidden.

The above consideration based on expressions (1) and (2) is only valid in the absence of the electron-electron interaction. The aim of this paper is to show that the Coulomb interaction changes the described I-V characteristic significantly. To begin with we discuss the particular kind of interaction to be accounted for.

Recently Glazman and Raikh[8] and Ng and Lee[9] considered the strong on-site Coulomb interaction between the electrons with different spin directions occupying the impurity level. It was shown that this interaction may lead to the Kondo anomalies in tunneling amplitude and increase the linear conductance as compared to the case of noninteracting electrons (3). The influence of on-site Coulomb interaction on current-voltage characteristics was also studied.[10,11] In particular, at $eV > \Gamma_L + \Gamma_R$ the Kondo resonance is destroyed and the only change in expression (4) is the additional factor $\frac{1}{2}$. It arises because strong on-site Coulomb repulsion allows only one of the two spin-degenerate states to be occupied.

The Coulomb interaction between the electrons in the leads also affects the tunneling, with its influence depending crucially on the dimensionality of the electron gas. In the 3D case the interaction gives rise to a small correction to the tunneling density of states;[12] in the following we neglect it. In the 2D disordered electron gas the first-order correction to the tunneling density of states with respect to the magnitude of interaction has a logarithmic singularity at small energies.[12] This correction depends on the degree of disorder and may also be neglected if the mean free path is large enough. Finally, in the 1D case the tunnel density of states is always suppressed at low energies,[13,14] so in this paper we restrict ourselves to the two- and three-dimensional cases.

The only remaining kind of interaction is that between the electron occupying the localized level and conduction electrons in the leads. In this paper we show that it gives rise to a new effect, namely, an increase of current $I(V)$ at small values of $V - V_{\rm th}$ qualitatively shown in Fig. 1 (solid line). The reasons for such behavior will be easier to understand if we suppose that the localized state is formed on the impurity with positive charge e (e.g., donor impurity in a semiconductor barrier).[15] In this case the total charge of the impurity and electron is zero and the electrons in the leads are not subjected to the external electric field as long as the localized state is occupied. However, at the moment when the electron tunnels from the impurity state to the right lead a positively charged hole appears on the impurity and produces an attractive potential U for the conduction electrons. Its influence on the tunneling process may be qualitatively understood by a simple calculation of the first-order correction to the tunneling amplitude with respect to the magnitude of the hole potential. Apart from the direct tunneling from the impurity level to the state with momentum k in the right lead described by the amplitude T_k, the electron can first tunnel into any other empty state q in the lead and then scatter to the state k by potential U. The total amplitude of such processes has the form

$$t_k = T_k + \sum_{q>k_F} T_q \frac{U_{qk}}{E_i - E_q} + \cdots, \tag{5}$$

where the ellipses signify the higher-order terms proportional to U^2, U^3, etc. Dropping the unessential dependence of the matrix elements on momenta we find that the sum over empty states gives rise to a logarithmic divergence in tunneling amplitude:

$$t = T\left(1 - \nu U \ln \frac{D}{E_i - E_F^R} + \cdots\right), \tag{6}$$

where ν is the density of states in the vicinity of the Fermi level in the right lead; D is some characteristic energy of the order of the bandwidth. Since the attractive potential created by the hole is described by negative U, the first-order correction is positive. Thus in formula (4) the tunneling rate Γ_R (proportional to the square of the amplitude t) acquires logarithmic dependence on $E_i - E_F^R$ or, in other terms, on $V - V_{\rm th}$:

$$\Gamma_R(V) \propto 1 + 2\nu|U| \ln \frac{D}{e(V - V_{\rm th})} + \cdots . \tag{7}$$

This change in Eq. (4) qualitatively corresponds to the solid line in Fig. 1.

It is important to note that the logarithmic correction appears due to the Coulomb interaction between the localized electron and conduction electrons. If it were not for this interaction, the hole potential would not be screened before the electron leaves the impurity. In this case we would have to take into account the change of the wave function of the conduction electrons due to their scattering on the impurity potential. This scattering leads to the excitation of virtual electon-hole pairs, and we would have to add to expression (5) the total amplitude

$$\sum_{q'<k_F} \frac{U_{q'k}}{E_k - E_{q'}} T_{q'}$$

of the processes in which at first the electron is scattered from any state q' below the Fermi level to the final state

k by potential U, and *then* the electron tunnels from the impurity level to state q'. This sum is negative and has logarithmic singularity at $E_i = E_k \to E_F^R$. So if we disregarded the electron-electron interaction, the logarithmic correction to the tunneling amplitude would have canceled. However, the Coulomb interaction of conduction electrons with the localized electron gives rise to the compensation of the impurity potential U by the potential $-U$ created by the localized electon. As a result the virtual electron-hole pairs are not excited.

In the following we will be mostly interested in the threshold behavior of the tunnel current $I(V)$, i.e., at $e(V - V_{\rm th}) \ll D$. [We can simultaneously suppose $e(V - V_{\rm th}) \gg \Gamma_L$, because the scale Γ_L at which the singularity must be smeared out is exponentially small.] In the vicinity of the threshold the first-order perturbation theory is not applicable and the higher-order terms in series (5) must be taken into account.

The simplest way to solve this problem is to use the analogy with the well-known Mahan-Nozières–De Dominicis (MND) problem in the theory of x-ray-absorption edge in metals (for recent reviews see Refs. 16 and 17). Contrary to our problem, in the MND theory the electron transition from a deep localized state to the vicinity of the Fermi level is stimulated by the x-ray photon. However, in the solution of the problem the photon plays no role. It only allows to satisfy the energy conservation law; the form of the matrix elements describing photon-assisted electron transitions does not affect the threshold behavior of the transition rate. On the other hand, the influence of the core-hole potential being switched on at the moment when the electron abandons the localized level is accounted for properly. Thus we can find the threshold behavior of $I(V)$ by applying the results of the MND theory.

The first result for the MND model was obtained by Mahan[18] who showed that the sum of the most divergent terms in the perturbation series for the transition rate has a power-law divergence at the threshold point. Applying this result to our problem we receive the I-V characteristic of the form

$$I(V) \propto \left(\frac{D}{e(V - V_{\rm th})}\right)^{\alpha} \theta(V - V_{\rm th}), \tag{8}$$

where $\alpha = 2\nu|U|$ and U represents the matrix element $U_{kk'}$ of the hole potential properly averaged over the directions of momenta k, k'.

The further investigations of the MND problem[19–21] showed that the threshold behavior of the transition rate can be found exactly. The exact theory also predicts the power-law edge singularity (8), with the exponent α being determined by the scattering phase shifts δ_χ for the conduction electrons with energy E_F on the core-hole potential:

$$\alpha = \frac{2\delta_0}{\pi} - \sum_\chi \left(\frac{\delta_\chi}{\pi}\right)^2 . \tag{9}$$

The sum in Eq. (9) must be taken over all the different scattering channels χ (including both spin directions); δ_0 is the maximum phase shift.[22] The two terms in Eq. (9) have different physical sense. The first one, $2\delta_0/\pi$, generalizes Mahan's original result $\alpha = 2\nu|U|$ and describes the increase of the tunneling rate due to the electron scattering on the core-hole potential. The second term in Eq. (9) is always negative; it was first predicted in the well-known work by Nozières and De Dominicis.[19] It accounts for the fact that the appearance of even a weak hole potential significantly modifies the many-particle wave function of the electron gas in the metal, so that it becomes almost orthogonal to the initial wave function.[23] Corresponding suppression of the transition rate gives rise to the second term in Eq. (9).

Thus the comparison of the tunneling problem with the MND problem shows that the I-V characteristic of the junction with a single impurity must have singular threshold behavior (8); the exponent α may be found with the aid of Eq. (9). The scattering problem to be solved is that for the potential induced in the metal by the charged impurity situated outside the metal.

We estimate the order of magnitude of the exponent α in Sec. II. A rigorous calculation of α for some particular kinds of the junction geometry is carried out in Secs. III and IV. The possibility of experimental observation of the increase in tunnel current in the vicinity of the threshold voltage and the modification of the formula (8) at $\Gamma_L \sim \Gamma_R$ are discussed in Sec. V.

II. QUALITATIVE ESTIMATES

The value of the exponent α is determined by the geometry of tunnel junction. In this section we estimate α in the simplest case when the right lead of the junction is a 2D electron gas, and the electron localized on a nearby impurity tunnels into it. Since the hole potential $U(\rho)$ is axially symmetric the scattering channels may be labeled by an integer quantum number m coinciding with the z component of the angular momentum (in units of $\hbar$). The electrons in channel m move in a potential

$$\frac{\hbar^2}{2M}\frac{m^2}{\rho^2} + U(\rho),$$

thus the electron with energy $E_F = \hbar^2 k_F^2/2M$ experiences the hole potential $U(\rho)$ only at $\rho \gtrsim m/k_F$. The characteristic size of potential $U(\rho)$ coincides with the distance d between the 2D layer and impurity, therefore the scattering phase shifts δ_m differ from zero only for $|m| \lesssim k_F d$. (Throughout this paper we suppose $k_F d \gg 1$.) The value of phase shifts may be estimated with the aid of the Friedel sum rule[24,19] which reads that the sum of all phase shifts equals πN, where N is the number of electrons screening the hole potential, $N = 1$. In our case

$$\sum_m \delta_m \sim (k_F d)\delta \sim 1,$$

that is, $\delta \sim (k_F d)^{-1}$. The second term in expression (9) may also be estimated,

$$\sum_m \left(\frac{\delta_m}{\pi}\right)^2 \sim (k_F d)\delta^2 \sim (k_F d)^{-1}, \tag{10}$$

and has the same order of magnitude. Thus the absolute value of the exponent α is of the order of $(k_F d)^{-1} \ll 1$. An analogous estimation for the 3D case gives $|\alpha| \sim (k_F d)^{-2}$, so at $k_F d \gg 1$ we always get the power-law singularity (8) in the I-V characteristic with a small exponent $|\alpha| \ll 1$.

The above estimate does not allow us to predict the sign of α because both terms in formula (9) have the same order of magnitude. Nevertheless, if all the phase shifts are positive [e.g., this is always the case for strictly attractive potential: $U(\rho) < 0$], we find

$$\sum_\chi \left(\frac{\delta_\chi}{\pi}\right)^2 = \left(\frac{\delta_0}{\pi}\right)^2 \sum_\chi \left(\frac{\delta_\chi}{\delta_0}\right)^2 < \left(\frac{\delta_0}{\pi}\right)^2 \sum_\chi \frac{\delta_\chi}{\delta_0} = \frac{\delta_0}{\pi^2} \sum_\chi \delta_\chi = \frac{\delta_0}{\pi}. \quad (11)$$

Here we again use the Friedel sum rule. The substitution of the last inequality into Eq. (9) gives $\alpha > \delta_0/\pi > 0$. Thus we can argue that the exponent α must be positive.

III. INFINITE 2D ELECTRODES

In this section we consider the most simple geometry of the junction when the exponent α allows rigorous calculation. Namely, we suppose that the electrodes are represented by infinite planes of 2D electron gas. In Sec. III A we take into account only the right electrode. The influence of the left electrode is considered in Sec. III B.

A. Tunneling into the plane of 2D electron gas

Suppose that the right electrode is a 2D electron gas situated in the plane $z = 0$, and impurity has the coordinates $x = y = 0$, $z = -d$. The following calculations are intended to illustrate the qualitative consideration of Sec. II, so we do not take into account the screening of the hole potential by the left electrode. The potential induced in a 2D electron gas by a charged impurity may be calculated in the Thomas-Fermi approximation[25] if the electron concentration is sufficiently high ($e^2/\hbar v_F \ll 1$). If the distance d is much larger than the Bohr radius a_B, the potential has the form

$$U(\boldsymbol{\rho}) = -\frac{\hbar^2}{2M} \frac{d}{(\rho^2 + d^2)^{3/2}}, \quad (12)$$

where $\boldsymbol{\rho} = (x, y)$ and M is the electronic mass. To get this result one can calculate the screening charge density $\sigma(\rho)$ within the electrostatic approximation, $\sigma(\rho) = -ed/2\pi(\rho^2 + d^2)^{3/2}$, and then apply the Thomas-Fermi approximation

$$\sigma(x, y) = \nu_2 e U(x, y) = e\frac{M}{\pi\hbar^2} U(x, y). \quad (13)$$

(Here $\nu_2 = M/\pi\hbar^2$ is the density of states in a 2D electron gas.) As we have seen in Sec. II, the scattering phase shifts may be labeled by the integer quantum number m and all of them are small: $\delta_m \ll 1$. This allows us to calculate the phase shifts in the Born approximation

$$\delta_m = -\pi M \int_0^\infty U(\rho) J_m^2(k_F \rho) \rho \, d\rho, \quad (14)$$

where $J_m(z)$ is the conventional Bessel function. Upon the substitution of potential (12) into the integral (14) the latter may be calculated at $k_F d \gg 1$ and the phase shifts have the form

$$\delta_m = \frac{1}{2} \frac{k_F d}{m^2 + (k_F d)^2}. \quad (15)$$

In correspondence with the estimates of Sec. II both terms in the right-hand side of formula (9) have the same order of magnitude:

$$\alpha = \frac{2\delta_0}{\pi} - 2 \sum_{m=-\infty}^{+\infty} \left(\frac{\delta_m}{\pi}\right)^2 = \frac{1}{\pi k_F d} - \frac{1}{4\pi k_F d} = \frac{3}{4\pi}(k_F d)^{-1}. \quad (16)$$

Here the factor 2 preceding the sum over m accounts for the spins of electrons.

B. The influence of the left electrode

So far we neglected the influence of the left electrode on the electron tunneling from impurity to the right electrode. This is a good approximation only if the distance d_L between the impurity and the left electrode is much larger than the distance d_R from the impurity to the right electrode. In the opposite case $d_L \lesssim d_R$ the presence of the left electrode is essential and leads to the decrease of the exponent α. For example, at $d_L \ll d_R$ the charge of the impurity is almost completely screened by the electrons of the left electrode and the potential induced in the right one is strongly suppressed. Thus the first term in the expression (9) is much less than our estimate of Sec. IV, i.e., $2\delta_0/\pi \ll (k_F d_R)^{-1}$. On the other hand, the sum over the scattering channels in the second term in Eq. (9) includes the scattering phase shifts in both electrodes and, corresponding to Eq. (10), may be estimated as $(k_F d_L)^{-1} \gg 2\delta_0/\pi$. Thus at $d_L \ll d_R$ the exponent α must be negative.[26]

To illustrate the above arguments we calculate α for the tunnel junction of a particular geometry: the two leads are represented by the planes of 2D electron gas, $z = -d_L$ and $z = d_R$, and the impurity is situated at point $x = y = z = 0$. The potential induced in the leads by the charged impurity is calculated in the Thomas-Fermi approximation in Appendix A. At $d_L, d_R \gg a_B$ the potential in the right electrode has the form

$$U_R(\rho) = -\frac{\hbar^2}{2M} \int_0^\infty \frac{\sinh q d_L}{\sinh q(d_L + d_R)} q J_0(q\rho) dq. \quad (17)$$

This potential scatters the electrons in the right lead; corresponding phase shifts may be calculated in the Born approximation by the substitution of Eq. (17) into Eq. (14). The result is

$$\delta_{mR} = \frac{\pi}{4k_F(d_L+d_R)} \frac{\sin\frac{\pi d_R}{d_L+d_R}}{\cosh\frac{\pi m}{k_F(d_L+d_R)} - \cos\frac{\pi d_R}{d_L+d_R}}. \tag{18}$$

The first term in the expression (9) for the exponent α is determined by the maximum of δ_{mR},

$$\frac{2\delta_0}{\pi} = \frac{1}{2k_F(d_L+d_R)} \cot\frac{\pi d_R}{2(d_L+d_R)}. \tag{19}$$

The second term in Eq. (9) is calculated by summing up the squares of the phase shifts in both electrodes, with δ_{mL} being obtained by replacement $L \leftrightarrow R$ in Eq. (18),

$$\sum_\chi \left(\frac{\delta_\chi}{\pi}\right)^2 = 2\sum_m \left\{\left(\frac{\delta_{mL}}{\pi}\right)^2 + \left(\frac{\delta_{mR}}{\pi}\right)^2\right\} = \frac{1}{4\pi k_F(d_L+d_R)}\left\{2 + \pi\frac{d_L-d_R}{d_L+d_R} \times \cot\frac{\pi d_R}{d_L+d_R}\right\}. \tag{20}$$

Subtracting Eq. (20) from Eq. (19) we find the dependence of the exponent α on the distances between impurity and electrodes:

$$\alpha = \frac{1}{2k_F(d_L+d_R)}\left\{\cot\frac{\pi d_R}{2(d_L+d_R)} - \frac{1}{\pi} + \frac{d_R-d_L}{2(d_L+d_R)}\cot\frac{\pi d_R}{d_L+d_R}\right\}. \tag{21}$$

In correspondence with our estimates, α is negative if the impurity is close to the left electrode $d_L < \beta d_R$, where $\beta \approx 0.40$, and positive otherwise.

IV. TUNNELING INTO THE CONFINED ELECTRODE

The case of tunneling into the infinite 2D electron gas considered in Sec. III may probably be realized in GaAs heterostructures. However the existing experiments[5–7] do not correspond to this geometry.

The experiments[5,6] were performed on silicon MOSFET's. The electodes were represented by the confined 2D electron gas, and contrary to the case of Sec. III the electrons tunnel from the impurity to the edge of the electrode, Fig. 2. To calculate the scattering potential

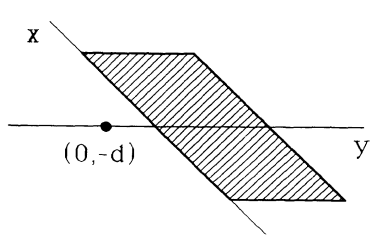

FIG. 2. The geometry of the 2D junction considered in Sec. IV B. The right electrode occupies the half-plane $y > 0$; impurity is situated at point $(0, -d)$. The left electrode is not shown.

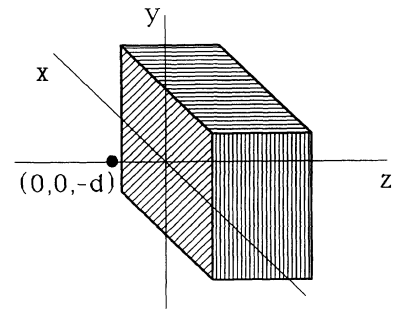

FIG. 3. The geometry of the 3D junction considered in Sec. IV C. The right electrode occupies the half-space $z > 0$, impurity is situated at point $(0, 0, -d)$. The left electrode is not shown.

for the electrons in the lead produced by charged impurity one can use the Thomas-Fermi approximation (13); the necessary density of screening charge may be found from the well-known solution of the electrostatic problem of a charge in the vicinity of a metal half-plane.[27] The resulting potential is

$$U(x,y) = -\frac{\hbar^2}{\pi M}\sqrt{\frac{d}{y}}\frac{1}{x^2+(d+y)^2}. \tag{22}$$

Unlike the potential (12) for the case of infinite 2D electron gas, this potential depends explicitly on the coordinates x, y and is not a function of single variable $\rho = \sqrt{x^2+y^2}$. Therefore the angular momentum m is no longer a quantum number and the evaluation of the phase shifts becomes more difficult.

The same difficulty arises in considering the case of 3D leads, Fig. 3, corresponding to the experiment in Ref. 7. The Thomas-Fermi calculation of the screened impurity potential inside the 3D metal is quite analogous to the two-dimensional case.[25] If the distance d between the impurity and metal surface is large as compared to the Debye radius κ^{-1}, the potential takes the form

$$U(\rho,z) = -\frac{2e^2 d}{\kappa}\frac{1}{(\rho^2+d^2)^{3/2}}e^{-\kappa z}. \tag{23}$$

Since $U(\rho, z)$ is not a central potential, the scattering problem may not be reduced to a one-dimensional one by using spherical coordinates.

A. Born approximation for the phase shifts in scattering on arbitrary potential

In this section we present a method of calculating the scattering phase shifts for a general potential. Let us reformulate our two- or three-dimensional problem in terms of a set of one-dimensional channels labeled by index q, e.g., the channels with definite components of angular momentum or the channels classified by the component q of momentum parallel to the edge of the lead. Suppose an electron in channel q with the energy E is scattered by the potential U. In general it may scatter to the states with energy E in all the channels q', and corresponding

amplitudes $S_{qq'}(E)$ constitute a unitary scattering matrix. In a special case of symmetric potential and properly chosen set of channels χ (e.g., central potential and the channels labeled by the components of angular momentum) the scattering matrix is diagonal:

$$S_{\chi\chi'}(E) = \delta_{\chi\chi'} \exp[2i\delta_\chi(E)], \tag{24}$$

where $\delta_\chi(E)$ is a scattering phase shift in channel χ, and $\delta_{\chi\chi'}$ is the Kronecker delta. In a general case of nonsymmetric potential we can diagonalize the scattering matrix by a unitary transformation, that is, to redefine the scattering channels in such a way that the electron with energy E in each of the channels χ is being scattered by potential U only to the same channel. Consider now a Hermitian matrix

$$\Delta_{qq'} = \frac{1}{2i}[\ln S(E)]_{qq'}. \tag{25}$$

Matrix $\Delta_{qq'}$ may be diagonalized by the same unitary transformation. The substitution of the diagonal scattering matrix (24) into definition (25) shows that the phase shifts δ_χ coincide with the eigenvalues of matrix $\Delta_{qq'}$.

We can now rewrite the expression (9) for the exponent α in terms of matrix $\Delta_{qq'}$. The first term $2\delta_0/\pi$ is expressed via the maximum eigenvalue of $\Delta_{qq'}$,

$$\delta_0 = \text{ maximum eigenvalue of } \Delta_{qq'}, \tag{26}$$

while the second one is proportional to the trace of Δ^2,

$$\sum_\chi \left(\frac{\delta_\chi}{\pi}\right)^2 = \frac{1}{\pi^2}\mathrm{Tr}\,\Delta^2 = \frac{1}{\pi^2}\sum_{qq'}|\Delta_{qq'}|^2. \tag{27}$$

The scattering phase shifts and, consequently, matrix $\Delta_{qq'}$ must be calculated for energy $E = E_F$.

Rigorous calculation of matrix $\Delta_{qq'}$ is hardly possible, but, as we have seen in Sec. II, at $k_F d \gg 1$ the scattering phase shifts are small and may be found in the Born approximation. Thus we can restrict ourselves to the calculation of matrix $\Delta_{qq'}$ in the first order of the perturbation theory with respect to the magnitude of scattering potential U. This calculation is straightforward, and the result is

$$\Delta_{qq'} = -\pi\sqrt{\nu_q\nu_{q'}}\,U_{qq'}. \tag{28}$$

Here $\nu_q = \nu_q(E)$ is the one-dimensional density of states in channel q; the matrix element $U_{qq'}$ must be calculated for the states with energy E in channels q and q'.

To our knowledge, the possibility of obtaining the threshold behavior of the transition rate in case of a general nonseparable potential with the aid of scattering matrix, Eqs. (25)–(27), has not been proven so far.[28] However, one can show (see Appendix B) that in the limit (28) of small phase shifts the formulas (9), (26), and (27) do give the correct result for the exponent α.

In Secs. IV B and IV C we apply the Born approximation (28) to the calculation of the exponent α for tunneling into confined electrodes, Figs. 2 and 3.

B. Two-dimensional electrode

Consider the case of tunneling into a 2D confined electrode, Fig. 2, corresponding to the experiments.[5,6] To calculate the exponent α in the Born approximation (26)–(28) we have to define a set of one-dimensional channels. It is convenient to choose the channels with definite x components of momentum $\hbar q$. The wave function of an electron in channel q with the energy $E_F > \hbar^2 q^2/2M$ has the form

$$\psi_q(x,y) = \sqrt{2}e^{iqx}\sin p_q y, \tag{29}$$

where $p_q = \sqrt{k_F^2 - q^2}$. The one-dimensional density of states ν_q is inversely proportional to p_q:

$$\nu_q = \frac{M}{\pi\hbar^2}\frac{1}{p_q}\theta(k_F - |q|). \tag{30}$$

The matrix element of potential (22) calculated with the wave functions ψ_q and $\psi_{q'}$ has the form

$$U_{qq'} = -\left(\frac{\pi}{2d}\right)^{1/2}\frac{\hbar^2}{M}e^{-|q-q'|d} \times\left(\frac{1}{\sqrt{|p_q - p_{q'}|}} - \frac{1}{\sqrt{p_q + p_{q'}}}\right). \tag{31}$$

(In fact, the square-root divergencies within the large parentheses disappear at small $|p_q \pm p_{q'}| \lesssim d^{-1}$. However, this will not affect the further calculations.) Now we are in a position to calculate the maximum phase shift δ_0 in the Born approximation (26) and (28). The phase shifts δ coincide with the eigenvalues of matrix (28); the corresponding integral equation may be written as follows:

$$\delta f(q) = \frac{1}{2\sqrt{2\pi d}}\int_{-k_F}^{k_F} dq'\,\frac{e^{-|q-q'|d}}{\sqrt{p_q p_{q'}}} \times\left(\frac{1}{\sqrt{|p_q - p_{q'}|}} - \frac{1}{\sqrt{p_q + p_{q'}}}\right) f(q'). \tag{32}$$

Due to the exponential factor the kernel of Eq. (32) has a sharp maximum at $q = q'$. Since it also increases at $|q| \to k_F$, we can find the maximum phase shift δ_0 by expanding the kernel over $(k_F - |q|)/k_F$. After some algebra we find that the maximum phase shift has the form

$$\delta_0 = 2^{-5/4}\pi^{-1/2}\Lambda_0\,(k_F d)^{-3/4}, \tag{33}$$

where Λ_0 is the maximum eigenvalue of the dimensionless integral equation

$$\Lambda\varphi(x) = \int_0^\infty dy\,e^{-|x^2-y^2|}\left(\frac{1}{\sqrt{|x-y|}} - \frac{1}{\sqrt{x+y}}\right)\varphi(y).$$

The last equation may be solved numerically, $\Lambda_0 \approx 1.66$.

Note, that contrary to the case of infinite 2D electrodes, we found here that $\delta_0 \sim (k_F d)^{-3/4}$ instead of $\delta_0 \sim (k_F d)^{-1}$. This difference appears due to the square-root singularity of the potential (22) in the vicinity of sample edge. The electrons with the momenta almost parallel to the edge experience greater value of the po-

tential and acquire greater phase shift. On the contrary, as we can see from Eq. (27) the second term in expression (9) for the exponent α is determined by all the possible directions of momentum, and the qualitative estimates of Sec. IV may be valid. The rigorous calculation of the sum (27) with the substitution of Eqs. (28), (30), and (31) confirms that the second term in Eq. (9) is of the order of $(k_F d)^{-1}$. Thus at $k_F d \gg 1$ the second term is negligible and

$$\alpha = 2^{-1/4}\pi^{-3/2}\Lambda_0\,(k_F d)^{-3/4} \approx 0.25(k_F d)^{-3/4}. \tag{34}$$

The combination of formulas (8) and (34) describes the current-voltage characteristic of the two-dimensional junctions.

C. Three-dimensional electrode

The case of the 3D electrode, Fig. 3, may be considered in a similar way. We define the set of one-dimensional channels with definite components $\hbar\mathbf{q}$ of momentum parallel to the electrode boundary. The wave functions of the electrons with energy E_F have the form

$$\psi_{\mathbf{q}}(\boldsymbol{\rho}, z) = \sqrt{2}e^{i\mathbf{q}\boldsymbol{\rho}}\sin p_q z. \tag{35}$$

The one-dimensional density of states in channel $\mathbf{q}$ is defined by the formula (30), matrix elements $U_{\mathbf{q}\mathbf{q}'}$ of potential (23) have the form

$$U_{\mathbf{q}\mathbf{q}'} = -16\pi e^2 \frac{p_q p_{q'}}{[\kappa^2 + (p_q - p_{q'})^2][\kappa^2 + (p_q + p_{q'})^2]} e^{-|\mathbf{q}-\mathbf{q}'|d}. \tag{36}$$

The substitution of Eqs. (30) and (36) into the Born approximation formula (28) gives the following integral equation for the scattering phase shifts:

$$\delta f(\mathbf{q}) = \frac{4Me^2}{\pi\hbar^2} \iint\limits_{q'<k_F} \frac{p_q p_{q'} \exp(-|\mathbf{q}-\mathbf{q}'|d)}{[\kappa^2 + (p_q - p_{q'})^2][\kappa^2 + (p_q + p_{q'})^2]} f(\mathbf{q}')d^2\mathbf{q}'. \tag{37}$$

Similar to the integral equation (32) for the 2D case, the kernel of Eq. (37) has a sharp exponential dependence on $|\mathbf{q}-\mathbf{q}'|$. However, contrary to Eq. (32), the kernel has the maximum at $q' = \sqrt{k_F^2 - \kappa^2/4} < k_F$, and this allows us to find the maximum eigenvalue in the limit of large d by replacement

$$\exp(-|\mathbf{q}-\mathbf{q}'|d) \to \frac{2\pi}{d^2}\delta(\mathbf{q}-\mathbf{q}').$$

In this approximation the eigenfunctions have the form $f(\mathbf{q}) = \delta(\mathbf{q}-\mathbf{q}_0)$, the maximum phase shift δ_0 corresponds to $q_0 = \sqrt{k_F^2 - \kappa^2/4}$ and equals

$$\delta_0 = \frac{2Me^2}{\hbar^2\kappa^3 d^2} = \frac{\pi}{2}\left(\frac{\pi\hbar v_F}{4e^2}\right)^{1/2}(k_F d)^{-2}, \tag{38}$$

where we substituted $(4e^2/\pi\hbar v_F)^{1/2}k_F$ for the inverse Debye radius κ.

We can also find the second term in expression (9) for the exponent α by substitution of Eqs. (28), (30), and (36) into Eq. (27). Corresponding calculation is straightforward and gives the result

$$\sum_{\chi}\left(\frac{\delta_\chi}{\pi}\right)^2 = \tfrac{1}{16}\ln\left(\frac{\hbar v_F}{e^2}\right)(k_F d)^{-2}.$$

The comparison with the first term $2\delta_0/\pi$ shows that the latter is much greater because of the parameter $\hbar v_F/e^2 \gg 1$ and the numeric coefficient. Thus in the 3D case we find the following value of the exponent:

$$\alpha = \left(\frac{\pi\hbar v_F}{4e^2}\right)^{1/2}(k_F d)^{-2} \tag{39}$$

in the expression (8) describing the threshold behavior of the current-voltage characteristic.

V. CONCLUSION

We considered the current-voltage characteristic of a tunnel junction with a single impurity situated inside the dielectric layer. The main result of the paper is the power-law threshold behavior (8), with the exponent α determined by Eqs. (16), (21), (34), or (39) depending on the junction geometry. The exponent α is small at $k_F d \gg 1$, therefore experimentally the most favorable situation corresponds to the leads fabricated of a material with small Fermi energy, e.g., heavily doped semiconductor. Another way to increase parameter $k_F d$ is to use dielectric layer with a wide forbidden gap, so that one could make small distance d between the impurity and the right lead. Note that at positive α the dependence (8) corresponds to negative differential conductance and may lead to a hysteresis of the current-voltage characteristic measured at fixed current.

In Sec. IV we neglected the influence of the left electrode on the tunneling into the right electrode. As we have seen in Sec. III, the left electrode screens the hole potential and reduces exponent α. The influence of the left electrode would weaken, if the impurity was situated closer to the right electrode; in this case to satisfy the condition $\Gamma_L \gg \Gamma_R$ one has to fabricate lower tunnel barrier on the left of the impurity. These conditions can be met in the experiments with scanning tunnel microscope,[29,30] where a small metal grain is separated from the substrate by an oxide layer, and there is a vacuum gap between the grain and microscope tip. In these experiments the exponent α must be very small because the characteristic size of the potential induced in the tip by a charged grain is of the order of grain size $r \gtrsim 100$ Å, i.e., $k_F r \gg 1$. However one would expect greater value of α if smaller grains were used.

Throughout this paper we supposed $\Gamma_L \gg \Gamma_R$, i.e., the

rate $2\Gamma_L/\hbar$ of the electron tunneling from the left lead to the impurity is much larger then the rate $2\Gamma_R/\hbar$ of tunneling from the impurity to the right lead. In this case the localized level is almost always filled and the expression for the current (8) describes the renormalization of Γ_R due to the scattering of conduction electrons by the charged hole. If Γ_L and Γ_R are comparable, the filling factor of the localized level $\Gamma_L/(\Gamma_L+\Gamma_R)$ will no longer be equal to 1. Corresponding decrease of the current is described by formula

$$I \propto \frac{\Gamma_L\Gamma_R(V)}{\Gamma_L+\Gamma_R(V)}\theta(V-V_{\rm th}),$$
$$\Gamma_R(V)=\Gamma_R^0\left(\frac{D}{e(V-V_{\rm th})}\right)^{\alpha}. \tag{40}$$

Note, for example, that at $\Gamma_L \ll \Gamma_R$ the current is mostly determined by Γ_L and any increase of Γ_R due to the Coulomb interaction leads to a small correction to the tunnel current.

The expressions (8) and (40) for the tunnel current are valid for the voltage not too close to the threshold value: $|V-V_{\rm th}| \gg \Gamma_L+\Gamma_R$. If $\Gamma_L \gg \Gamma_R$, it is possible also to find $I(V)$ for $|V-V_{\rm th}| \sim \Gamma_L+\Gamma_R$. In this case one has to take into account the finite lifetime of the hole due to the electron tunneling from the left lead. To do this we must add the imaginary part $-i\Gamma_L$ to the energy E_i of the localized electron in formulas (B3) and (B13) of Appendix B. The I-V characteristic for small $\alpha \ll 1$ has the form

$$I \propto \left(\frac{D}{\sqrt{E_i^2+\Gamma_L^2}}\right)^{\alpha}\left(\frac{\pi}{2}+\arctan\frac{E_i}{\Gamma_L}\right), \tag{41}$$

where the energy of the impurity E_i (counted from E_F^R) depends on the voltage: $E_i=\lambda e(V-V_{\rm th})$, the numeric factor λ being determined by the geometry of the junction.

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APPENDIX A: POTENTIAL INDUCED BY AN IMPURITY IN TWO PARALLEL 2D ELECTRODES

Suppose the two layers of 2D electron gas are situated in planes $z=-d_L$, $z=d_R$, and a charged impurity has the coordinates $x=y=z=0$. Following the review by Ando, Fowler, and Stern,[25] we find the electrostatic potential $\phi(\rho,z)$ from Poisson's equation

$$\epsilon\Delta\phi = -4\pi e\delta(\boldsymbol{\rho})\delta(z) - 4\pi\sigma_L(\boldsymbol{\rho})\delta(z+d_L) - 4\pi\sigma_R(\boldsymbol{\rho})\delta(z-d_R), \tag{A1}$$

where ϵ is the dielectric constant; the charge density $\sigma_{L(R)}(\boldsymbol{\rho})$ induced by the impurity in the left (right) electrode in the Thomas-Fermi approximation is connected with the potential

$$\sigma_L(\boldsymbol{\rho}) = -\frac{Me^2}{\pi\hbar^2}\phi(\boldsymbol{\rho},-d_L), \quad \sigma_R(\boldsymbol{\rho}) = -\frac{Me^2}{\pi\hbar^2}\phi(\boldsymbol{\rho},d_R). \tag{A2}$$

After the substitution of Eq. (A2) into Eq. (A1) and a Fourier transformation of $\phi(\boldsymbol{\rho},z)$ over the transverse coordinate $\boldsymbol{\rho}$, we obtain a simple differential equation

$$\frac{d^2}{dz^2}\phi(\mathbf{q},z) - q^2\phi(\mathbf{q},z) = -\frac{4\pi e}{\epsilon}\delta(z) + \frac{4Me^2}{\epsilon\hbar^2}\delta(z+d_L)\phi(\mathbf{q},-d_L) + \frac{4Me^2}{\epsilon\hbar^2}\delta(z-d_R)\phi(\mathbf{q},d_R). \tag{A3}$$

The solution of this equation allows us to determine the potential $U_R(\rho)=-e\phi(\rho,d_R)$ induced by the impurity in the right electrode

$$U_R(\rho) = -\frac{\hbar^2}{2M}\int_0^\infty \frac{\sinh qd_L + \frac{1}{4}qa_Be^{qd_L}}{\sinh q(d_L+d_R) + (1+\frac{1}{4}qa_B)\frac{1}{2}qa_Be^{q(d_L+d_R)}}\, qJ_0(q\rho)dq, \tag{A4}$$

where we introduced the Bohr radius $a_B=\epsilon\hbar^2/Me^2$. At $d_L, d_R \gg a_B$ the parameter $qa_B \sim a_B/d_{L(R)}$ is small, and we obtain the expression (17). The potential in the left electrode coincides with Eq. (A4) after the replacement $L \leftrightarrow R$.

APPENDIX B: LEADING LOGARITHM APPROXIMATION FOR TUNNELING RATE

The expression (9) for the exponent α in formula (8) for the tunnel current is obtained as a result of the MND theory in the case of a separable potential. Here we calculate α in the first nonvanishing order of the perturbation theory with respect to the magnitude of an arbitrary nonseparable potential.

We describe the tunneling of electron from impurity to the right electrode by the Hamiltonian

$$H = H_0 + H_T,$$
$$H_0 = \sum_k \epsilon_k a_k^\dagger a_k + E_i b^\dagger b + \sum_{kk'} U_{kk'} a_k^\dagger a_{k'} b b^\dagger,$$
$$H_T = \sum_k (T_k a_k^\dagger b + T_k^* b^\dagger a_k).$$

Here ϵ_k, $a_k^\dagger$ are energy and creation operator of electron in the right lead, $b^\dagger$ is the creation operator of the electron occupying impurity level, and $U_{kk'}$ is the matrix element of the potential induced in the right lead by the

charged impurity. Hamiltonian H_T describes the tunneling of electrons between impurity and electrode; T_k is corresponding matrix element. Due to the presence of the left electrode and the condition $\Gamma_L \gg \Gamma_R$ the occupation number of the impurity level is determined by the Fermi level E_F^L in the left electrode. Thus the impurity level is filled at $E_i > 0$ (the energy origin is chosen at Fermi level E_F^R), and we can calculate the current in the junction as

$$I = -e\left\langle \frac{d}{dt} b^\dagger b \right\rangle = ie\langle [b^\dagger b, H_T]\rangle = -2e\,\mathrm{Im}\sum_{k'} T_{k'}^* \langle b^\dagger a_{k'}\rangle.$$

We now calculate the average $\langle b^\dagger a_{k'}\rangle$ in the first-order perturbation theory with respect to the matrix element T_k and find

$$I = 2e\,\mathrm{Re}\int_{-\infty}^{0} \sum_{kk'} T_k T_{k'}^* F_{kk'}(t)dt, \tag{B1}$$

$$F_{kk'}(t) = \langle 0|T\left\{a_k^\dagger(t) b(t) b^\dagger(0) a_{k'}(0)\right\}|0\rangle. \tag{B2}$$

Here T is the time ordering operator; $a_k(t), b(t)$ are Heisenberg operators corresponding to Hamiltonian H_0; the averaging in Eq. (B2) is performed over the initial state with occupied impurity level and filled Fermi surface.

The quantity $F_{kk'}(t)$ may be studied by expanding Eq. (B2) in diagrammatic series over the interaction $U_{kk'}$. Some general properties of this series have been established by Nozières and De Dominicis[19] who showed that $F_{kk'}$ may be written in the form

$$F_{kk'}(t) = e^{-iE_i t} L_{kk'}(t) e^{C(t)}. \tag{B3}$$

Here $L_{kk'}(t)$ is the open line contribution[19] describing the scattering of the electron tunneled from the impurity; $C(t)$ is the sum of all single loops and describes the adjustment of the Fermi sea to the hole potential. These two factors give rise to the two terms in Eq. (9). Below we calculate both contributions in Eq. (9) in the first nonvanishing order with respect to $U_{kk'}$.

The first nontrivial contribution to $C(t)$ is due to the two-vertex loop diagram. Its calculation is straightforward[16] and gives

$$C_2(t) = i\Delta E_i t - \sum_{kk'} |U_{kk'}|^2 \theta(\epsilon_k)\theta(-\epsilon_{k'}) \frac{1 - e^{i(\epsilon_k - \epsilon_{k'})t}}{(\epsilon_k - \epsilon_{k'})^2}. \tag{B4}$$

After the substitution into Eq. (B3) the first term gives only an unessential renormalization of energy E_i, while the second one is proportional to $\ln t$ and gives rise to the power law in Eq. (B3). It is shown in Ref. 19 that this power-law dependence is responsible for the second term in Eq. (9). To evaluate it we suppose that each state k is characterized by the channel index q and one-dimensional momentum p. Then at $t \to -\infty$ the second term in Eq. (B4) has the form

$$-\sum_{qq'} \nu_q \nu_{q'} |U_{qq'}|^2 \int_0^\infty d\epsilon_k \int_{-\infty}^0 d\epsilon_{k'}\, e^{-(|\epsilon_k| + |\epsilon_{k'}|)/D} \frac{1 - e^{i(\epsilon_k - \epsilon_{k'})t}}{(\epsilon_k - \epsilon_{k'})^2} = -\frac{1}{\pi^2}\mathrm{Tr}\Delta^2 \ln(-iDt), \tag{B5}$$

where we introduced a cutoff D of the order of the bandwidth; matrix Δ is defined by Eq. (28). As we expected, the factor preceding $\ln(-iDt)$ coincides with the second term in Eq. (9) obtained by the substitution of approximation (28) into expression (27).

The first term in Eq. (9) originates from the open line contribution $L_{kk'}(t)$. To determine it we have to find the asymptotic behavior at $t \to -\infty$ of the quantity

$$L(t) = \sum_{kk'} T_k T_{k'}^* L_{kk'}(t) = \sum_{n=0}^{\infty} L_n(t), \tag{B6}$$

where $L_n(t)$ is the contribution of the open line diagram with n vertices,

$$L_n(t) = i\sum_{kk'} T_k T_{k'}^* \sum_{k_1 \cdots k_{n-1}} U_{k'k_1} U_{k_1 k_2} \cdots U_{k_{n-1}k} \times \int_t^0 \cdots \int_t^0 dt_1 \cdots dt_n G_{k'}(-t_1) G_{k_1}(t_1 - t_2) \cdots G_{k_{n-1}}(t_{n-1} - t_n) G_k(t_n - t).$$

Here $G_k(t)$ is a conventional unperturbed Green's function. We now again substitute $k = (q, p)$. At $t \to -\infty$ only the electronic states with energies $\epsilon_k = \epsilon_{qp} \to 0$ contribute to $L_n(t)$, hence within the leading logarithm approximation we can neglect the dependence of matrix elements on p and substitute $U_{kk'} = U_{qq'}$ and $T_k = T_q$. After this we find

$$L_n(t) = \sum_{qq'} T_q T_{q'}^* \sum_{q_1 \cdots q_{n-1}} \gamma_n U_{q'q_1} U_{q_1 q_2} \cdots U_{q_{n-1}q}, \tag{B7}$$

$$\gamma_n = i\int_t^0 \cdots \int_t^0 dt_1 \cdots dt_n G_{q'}(-t_1)G_{q_1}(t_1-t_2)\cdots G_{q_{n-1}}(t_{n-1}-t_n)G_q(t_n-t). \tag{B8}$$

Here we introduced the Green's functions summed over p,

$$G_q(t) = \sum_p G_{qp}(t) = -\frac{\nu_q}{t - iD^{-1}\mathrm{sgn}\, t}, \quad Dt \to -\infty.$$

At $t \to -\infty$ the integral (B8) diverges logarithmically. The leading term of its asymptotics has the form

$$\gamma_n = \nu_{q'}\nu_{q_1}\cdots\nu_{q_{n-1}}\nu_q(-it)^{-1}\frac{(-2)^n}{n!}\ln^n(-iDt). \tag{B9}$$

The substitution of Eq. (B9) into Eq. (B7) gives

$$L_n(t) = \sum_{qq'} T_q T^*_{q'}\sqrt{\nu_q\nu_{q'}}\,(-it)^{-1}\frac{1}{n!}\left\{\left(\frac{2}{\pi}\Delta\right)^n\right\}_{qq'} \times \ln^n(-iDt),$$

where the matrix $\Delta_{qq'}$ is defined by formula Eq. (28). Now we can rewrite the sum over n in Eq. (B6) as

$$L(t) = \sum_{qq'} T_q T^*_{q'}\sqrt{\nu_q\nu_{q'}}\,(-it)^{-1} \times \left\{\exp\left[\frac{2}{\pi}\Delta\ln(-iDt)\right]\right\}_{qq'}. \tag{B10}$$

In principle, one can find the set of eigenvalues δ_χ and corresponding eigenvectors $f_\chi(q)$ of the Hermitian matrix $\Delta_{qq'}$. Then the expression Eq. (B10) takes the form

$$L(t) = \sum_\chi |\tau_\chi|^2(-it)^{-1}\exp\left[\frac{2}{\pi}\delta_\chi\ln(-iDt)\right], \tag{B11}$$

$$\tau_\chi = \sum_q \sqrt{\nu_q}\,T_q f_\chi(q). \tag{B12}$$

Finally, we can use Eqs. (B11) and (B5) to calculate the current (B1) in the junction

$$\begin{aligned} I &= 2e\,\mathrm{Re}\int_{-\infty}^0 e^{-iE_it}L(t)\,e^{C(t)}dt \\ &= 2e\sum_\chi |\tau_\chi|^2\,\mathrm{Re}\int_{-\infty}^0 e^{-iE_it}(-it)^{-1} \\ &\qquad \times(-iDt)^{2\delta_\chi/\pi-\mathrm{Tr}\Delta^2/\pi^2}dt. \end{aligned} \tag{B13}$$

If the scattering phase shifts δ_χ are small, the calculation of the integral in Eq. (B13) gives

$$I = 2\pi e\sum_\chi |\tau_\chi|^2\left(\frac{D}{E_i}\right)^{2\delta_\chi/\pi-\mathrm{Tr}\Delta^2/\pi^2}. \tag{B14}$$

At $E_i \propto e(V - V_{\mathrm{th}}) \to 0$ the leading contribution to the current is associated with the channel $\chi = 0$ providing the maximum phase shift δ_0. Neglecting the contributions of other channels, we receive the threshold behavior (8) with the exponent α determined by Eq. (9).

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