Coulomb Blockade of Tunneling into a Quasi-One-Dimensional Wire

K. A. Matveev[(a)] and L. I. Glazman
Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455
(Received 15 September 1992)

Tunneling into the correlated electron system in a quantum wire is considered. For the first time, the problem is solved for an arbitrary number of transverse electron modes in the wire. The tunnel density of states has power-law asymptotics at low energies, $\nu(\epsilon) \propto \epsilon^{\alpha}$. A simple formula expressing α in terms of velocities of soundlike excitations (plasmons) propagating along the wire is obtained and the eigenvalue equation for these velocities is found. The general result for α allows one to extrapolate correctly between the known cases of a one-mode wire and a wide three-dimensional wire.

PACS numbers: 73.20.Dx, 73.40.Gk

It is well known that Coulomb blockade in a single junction is smeared due to quantum fluctuations of charge in the leads (see, e.g., a review [1]). These fluctuations were considered [2–4] by means of a phenomenological model in which the electron after tunneling through a junction interacts with the electromagnetic modes of the environment. Such a model adequately describes tunneling into a wide metal wire and yields the power law for differential conductance:

$$\frac{dI}{dV} \propto V^{\alpha}, \quad V \to 0. \tag{1}$$

Here the exponent α is determined by the zero-frequency impedance $Z(0)$ of the wire (i.e., "environment"),

$$\alpha = \frac{e^2}{\pi\hbar} Z(0). \tag{2}$$

The width of a metal wire normally is much larger than the Fermi wavelength. The opposite limit can be realized in semiconductor heterostructures, where it is possible to produce a wire carrying only one or a few transverse electron modes by means of a strong lateral confinement of the two-dimensional electron gas [5]. Microscopic theory of tunneling into a one-mode interacting electron gas was developed recently [6, 7]. It follows from [6] that the power-law dependence (1) is still valid for tunneling into the edge of a one-dimensional (1D) wire. However, the exponent α is equal to

$$\alpha = \frac{1}{\pi\hbar}\sqrt{m/n\kappa} - 1. \tag{3}$$

Here m is the effective electron mass, n is 1D electron density in the wire, and κ is the compressibility of interacting electron gas.

In the experiments with semiconductor heterostructures the width of the wire may be changed by adjusting the potential on the gates [5]. This gives rise to the interesting question of the generalization of power-law dependences (1)–(3) in the case of tunneling into a few-mode wire. It is important to note that the straightforward application [8] of formula (2) to this problem gives an incorrect result. In particular, tunneling into a one-mode wire with noninteracting electrons must be described by a constant density of states near the Fermi level, and there is no singularity in dI/dV [9]. On the other hand, the substitution of the corresponding impedance $Z(0) = 2\pi\hbar/e^2$ into (2) leads [8] to the unreasonable result $\alpha = 2$. In this paper we develop a consistent theory of tunneling into a multimode wire. This nontrivial problem can be solved analytically for the case of a long-range interaction between electrons. Fortunately this approximation adequately describes a wire formed in a semiconductor heterostructure. As a result of the long range of the interaction electron-electron scattering with large transfer of momentum is suppressed. This allows us to propose a simple model of a multimode electron liquid, generalizing the well-known Luttinger model of a one-dimensional interacting electron gas. Our theory is valid for any number N of modes and reproduces the correct limits (2) and (3) for $N = 1$ and $N \to \infty$.

The differential conductance $dI/dV \propto \sum_i |t_i|^2 \nu_i(eV)$ is determined by the amplitude t_i of tunneling into the i-th mode and the tunnel density of states in this mode $\nu_i(\omega)$. The latter can be expressed in terms of the Green function,

$$\nu_i(\omega) = \mathrm{Re}\frac{1}{\pi}\int_0^{\infty} e^{i\omega t}\langle\psi_i(t)\psi_i^{\dagger}(0)\rangle dt, \tag{4}$$

where operator $\psi_i^{\dagger}$ creates an electron near the edge of the wire in the i-th mode.

To calculate the Green function in Eq. (4) we employ the bosonization technique [10]. The Hamiltonian of the wire $H = H_0 + H_{\mathrm{int}}$ consists of a free electron part H_0 and the electrostatic interaction H_{int}. In boson representation,

$$H_0 = \sum_{i=1}^{N}\int_0^{\infty}\left(\frac{p_i^2(x)}{2\rho_i} + \frac{1}{2}\rho_i v_i^2 (u_i')^2\right) dx, \tag{5}$$

where $\rho_i = mn_i$ is the density of electron liquid in the

 © 1993 The American Physical Society

i-th mode, $u_i(x)$ is the displacement operator of this liquid, and $p_i(x)$ is the conjugate momentum density operator, $[u_i(x), p_j(y)] = i\hbar\delta_{ij}\delta(x-y)$. Velocity v_i is proportional to the electron density in the i-th mode, $v_i = \pi\hbar n_i/m$, and equals the Fermi velocity in the noninteracting 1D electron gas at this density. Electron-electron interaction with potential $V(x-y)$ is described by the Hamiltonian

$$H_{\rm int} = \frac{1}{2}\sum_{i,j=1}^{N}\int_0^\infty\int_0^\infty n_i u_i'(x) n_j u_j'(y) V(x-y)\,dy\,dx. \tag{6}$$

Products $n_i u_i'(x)$ have the simple meaning of the deviations of the electron density at point x from its average value n_i. The behavior of the tunnel density of states ν_i at small ω is determined by long-wavelength quantum fluctuations of the fields $u_j(x)$. To consider these long-wavelength fluctuations, one can substitute $V(x-y) = V_0\delta(x-y)$ into (6) and obtain

$$H_{\rm int} = \frac{1}{2}V_0\sum_{i,j=1}^{N} n_i n_j \int_0^\infty u_i'(x)u_j'(x)dx, \tag{7}$$

where $V_0 = \int V(x)dx$ is the zero-momentum Fourier component of the interaction potential. For the most realistic case of a wire formed by the gate-induced depletion, Coulomb potential $V(x) = e^2/\varepsilon x$ is screened at $x \gtrsim D$, and $V_0 = 2(e^2/\varepsilon)\ln k_F D$. (Here k_F is the Fermi wave vector of electrons in the wire, D is the distance between the wire and the gate, $k_F D \gg 1$; we assume that the wire and the gate are separated by a medium with dielectric constant ε.)

Potential $V(x)$ in the Hamiltonian $H_{\rm int}$ describes the interaction between charge densities in two different cross sections of the wire. We neglect the matrix elements of interaction that cause intermode transitions. These elements correspond to the Fourier components of the potential with nonzero wave vector $k \sim k_F$, and are small for a smooth long-range potential. In the case of Coulomb potential, the intermode elements do not contain a large logarithmic factor $\ln k_F D$. The same parameter allows us to neglect the backscattering of electrons within one spin-degenerate mode: it is well known [11] that for repulsive interaction weak backscattering leads only to a small renormalization of interaction constants.

The quadratic Hamiltonian (5),(7) can be transformed to a diagonal form,

$$H = \sum_{l=1}^{N}\int_0^\infty\left(\frac{1}{2m}P_l^2(k) + \frac{1}{2}m s_l^2 k^2 Q_l^2(k)\right)dk, \tag{8}$$

and describes N soundlike modes with the velocities s_l. The initial variables $u_i(x)$, $p_i(x)$ are related to the new ones by a unitary transformation,

$$u_i(x) = \frac{1}{\sqrt{n_i}}\sum_{l=1}^{N}\gamma_{il}\int_0^\infty \phi_k(x)Q_l(k)dk, \tag{9}$$

$$p_i(x) = \sqrt{n_i}\sum_{l=1}^{N}\gamma_{il}\int_0^\infty \phi_k(x)P_l(k)dk. \tag{10}$$

Here $\phi_k(x) = \sqrt{2/\pi}\sin kx$; the sound velocities s_l and the components of orthogonal matrix γ_{il} are determined by the following eigenvalue problem:

$$\sum_{j=1}^{N}\left(v_i^2\delta_{ij} + \sqrt{v_i v_j}\frac{V_0}{\pi\hbar}\right)\gamma_{jl} = s_l^2\gamma_{il}. \tag{11}$$

To calculate the tunnel density of states (4) we have to express the fermion creation operator $\psi_i^\dagger$ via boson variables u_i and p_i. We adopt the following definition:

$$\psi_i^\dagger = \exp\left(-\frac{i}{\hbar n_i}\Pi_i\right), \tag{12}$$

where

$$\Pi_i = \int_0^\infty p_i(x)dx. \tag{13}$$

Expression (12) for the creation operator has an obvious meaning of a shift operator of all the electron liquid in mode i by a distance n_i^{-1} towards the wire edge. Such a shift of electron liquid corresponds to formation of extra charge density $en_i u_i' = e\delta(x)$ at the edge and describes the charge of a created electron. Formula (12) essentially coincides with the standard definition for the electron creation operator obtained in the bosonization technique [10]. The latter definition contains an extra factor $\exp(i\pi n_i u_i)$ which equals 1 due to the boundary condition $u_i = 0$ at the edge of the wire.

Since the Hamiltonian (5),(7) is quadratic, the electron Green function in Eq. (4) now can be easily found. The method is analogous to the one used in the calculation of the Debye-Waller factor for the Mössbauer effect [12]. The density of states (4) takes the form

$$\nu_i(\omega) = {\rm Re}\frac{1}{\pi}\int_0^\infty e^{i\omega t}\exp\left\{-\sum_{l=1}^{N}\gamma_{il}^2\frac{s_l}{v_i}\int_0^\infty\frac{dk}{k}\left(1-e^{-is_l kt}\right)\right\}dt. \tag{14}$$

The integral over k in Eq. (14) diverges logarithmically at the upper limit. The cause is that Hamiltonian (8) was derived in the long-wavelength approximation and is valid only at k smaller than the inverse of the characteristic radius r_0 of interaction potential $V(x-y)$. So we have to introduce a cutoff factor $e^{-r_0 k}$ into the integral over k in Eq. (14), in complete analogy with the well-known theory of a single-mode Luttinger liquid [13]. Similarly to Eq. (1), we find from (14) for the multimode case $\nu_i(\omega) \propto \omega^{\alpha_i}$, with the exponent

$$\alpha_i = \sum_{l=1}^{N} \gamma_{il}^2 \frac{s_l}{v_i} - 1. \tag{15}$$

This equation is the central result of our paper. It reduces the problem of low-energy asymptotics of the tunnel density of states to a simple eigenvalue problem (11).

For practical purposes, it is interesting to determine the dependence of α_i on the continuously varying one-dimensional density of electrons. In experiments the tunneling of gate voltage [5, 14] allows one to increase to 1D density n by continuous widening of the channel or by increasing the Fermi energy E_F. For any reasonable number of modes the eigenvalue problem (11) can be easily solved numerically. An example of numerical results for $\alpha_i(E_F)$ is shown in Fig. 1. We used a model of wire with electron confinement in the transverse direction caused by a parabolic potential

$$U(y) = \frac{m\Omega^2 y^2}{2},$$

and plotted α_i as a function of the dimensionless variable $E = E_F/\hbar\Omega - 1/2$.

To understand the nature of singularities in Fig. 1, we consider analytically the case of a one-mode wire corresponding to the interval $0 < E < 1$. Solution of the eigenvalue problem (11) is obvious: $\gamma_{11} = 1$, $s_1 = (v_1^2 + v_1 V_0/\pi\hbar)^{1/2}$. Substitution of $\gamma_{11} = 1$ into Eq. (15) gives

$$\alpha_1 = \frac{s_1}{v_1} - 1. \tag{16}$$

At small E_F, both v_1 and $s_1 \propto v_1^{1/2}$ are small, and $\alpha_1 \propto v_1^{-1/2} \to \infty$. This corresponds to the singularity $\alpha_1 \propto E^{-1/4}$ at $E \to 0$ in Fig. 1. Note also that with the help of the relation $s_1 = \sqrt{n_1/m\kappa}$ between sound velocity and compressibility Eq. (16) reproduces the result (3) of Kane and Fisher [6].

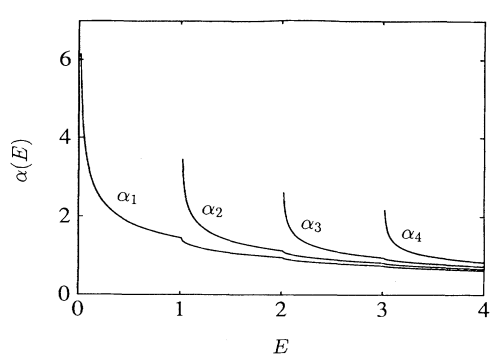

FIG. 1. Exponents α_i describing the low-energy tunnel density of states, $\nu_i \propto \epsilon_i^{\alpha}$, for different modes i. Dependence of α_i on Fermi energy is shown for a wire formed by potential $U(y) = m\Omega^2 y^2/2$ confining electrons in the transverse direction, $E = E_F/\hbar\Omega - 1/2$. The singularities at $E = 0, 1, 2, \ldots$ originate from switching of new modes. The interaction measured in dimensionless units $(V_0/\pi\hbar)(m/2\hbar\Omega)^{1/2}$ for this plot equals 5.

To study analytically other peculiarities in Fig. 1 and find the relation between (15) and the result (2) for a wide wire, we proceed further with solving the eigenvalue problem (11), which is possible due to separability of the interaction term. The equation for eigenvalues s_l^2 can be easily found:

$$\sum_{i=1}^{N} \frac{v_i}{s_l^2 - v_i^2} = \frac{\pi\hbar}{V_0}. \tag{17}$$

Matrix elements γ_{il} can be expressed in terms of the set of velocities v_i directly related to the electron densities in the corresponding modes, $v_i = \pi\hbar n_i/m$, and the set of sound velocities s_l determined by Eq. (17),

$$\gamma_{il}^2 = \frac{v_i}{(s_l^2 - v_i^2)^2} \left[\sum_{j=1}^{N} \frac{v_j}{(s_l^2 - v_j^2)^2} \right]^{-1}. \tag{18}$$

For a given set of densities n_i, solution of (17) is equivalent to finding of all N zeros of a polynomial. In the case of two modes explicit analytical expressions for α_1 and α_2 can be easily found from Eqs. (15), (17), and (18). We give here only formulas for two limits, when the results can be presented in a compact form. We start from the case $v_1 \to v_2$ that corresponds, e.g., to a single transverse mode with spin degeneracy. In this case,

$$\gamma_{il}^2 = 1/2, \quad s_1 = (v_1^2 + 2v_1 V_0/\pi\hbar)^{1/2}, \quad s_2 = v_1.$$

(The result for the sound velocity s_1 coincides with the well-known expression for the plasmon velocity in the Tomonaga model, see, e.g., [15].) It now follows from (15) that

$$\alpha_1 = \alpha_2 = \tfrac{1}{2}[(1 + 2V_0/\pi\hbar v_1)^{1/2} - 1].$$

This formula relates the exponents directly to the interaction potential V_0. Previously α was expressed [16] in terms of phenomenological spin and charge conductances of the 1D Luttinger liquid.

In the limit of strong interaction, $V_0 \gg \pi\hbar(v_1 + v_2)$, Eq. (18) can also be solved easily, and one finds

$$\alpha_1 = \left(\frac{V_0}{\pi\hbar(v_1 + v_2)} \right)^{1/2}, \tag{19}$$

$$\alpha_2 = \left(\frac{V_0}{\pi\hbar(v_1 + v_2)} \right)^{1/2} + \frac{v_1}{v_1 + v_2} \sqrt{\frac{v_1}{v_2}}. \tag{20}$$

The exponent α_2 diverges at $v_2 \to 0$, similarly to the behavior of α_1 observed for the case of a single mode. It is interesting to note, however, that the coefficient in the singular part of α_2 is parametrically smaller than the proper coefficient in α_1, see Eq. (16), and does not depend on V_0. This is due to the effect of screening

of the Coulomb interaction in the second mode by the electrons of the first mode. Obviously, screening acts in the same way and leads to a similar effect for the higher modes (as illustrated by Fig. 1). Note also that at small v_2 the expression (19) for α_1 differs from its value $v_2 = 0$ (when the second mode switches on) by $\delta\alpha_1 \approx -(\alpha_1/2v_1)v_2 \propto -\sqrt{E-1}$. This square-root peculiarity in $\alpha_1(E)$ and similar peculiarities at $E = 2, 3, \ldots$ are clearly seen in Fig. 1.

Now we consider the case of a wide wire in which the number of transverse modes is large: $N \gg 1$. Our goal is the calculation of α_i with the accuracy $O(1/N)$. First, we use the unitarity of matrix γ_{il} to rewrite the result (15) as follows:

$$\alpha_i = \sum_{l=1}^{N} \gamma_{il}^2 \left(\frac{s_l}{v_i} - 1 \right). \tag{21}$$

In the limit $N \to \infty$ the largest of the sound velocities, $s_1 = (nV_0/m)^{1/2} \propto \sqrt{N}$, grows infinitely while the others remain finite: $v_l < s_l < v_{l-1}$ for $l > 1$. [This directly follows from the dispersion relation (18) for the velocities.] So we extract the term corresponding to $l = 1$ in sum (21) and consider the remaining terms separately. Since the matrix elements (20) decay rapidly at $|i - l| \gg 1$, only a few of these terms with $|i - l| \sim 1$ should be taken into account. Each of them can be estimated as $\gamma_{il}^2 |s_l - v_i|/v_i \sim 1/N$. Thus with the accuracy of $O(1/N)$ only the term with $l = 1$ contributes to the sum (21). Using the value of γ_{i1} in the limit $s_1 \gg v_F$, we find

$$\alpha_i = \frac{s_1}{\sum v_j} = \frac{m s_1}{\pi \hbar n}. \tag{22}$$

One can easily see that $\alpha_i \sim (V_0/\pi\hbar v_F N)^{1/2}$; i.e., we did not exceed the accuracy $1/N$ of the calculation. Equation (22) shows that for a wide wire the exponent α_i is determined only by the velocity of macroscopic plasmon s_1 and one-dimensional density of electrons n. This result coincides with formula (2) obtained in macroscopic approach [3, 4], because $Z(0) = m s_1/ne^2$. [One can see this by comparing the energy $I^2 Z(0)$ dissipated by current $I = nev$ in a half-infinite wire with the energy $mnv^2 s_1$ of emitted plasmons.]

Comparison of Eq. (17) with Eq. (22) shows that the exponents α being expressed in terms of the plasmon velocity s_1 and total 1D electron density n differ by 1 in the cases of one-mode and wide wires. The difference is due to the fact that in a wide wire, along with a plasmon, there are also excitations with velocities of order of v_F. These excitations represent Fermi-liquid quasiparticles. Therefore, the tunneling electron creates not only a plasmon, but also quasiparticles. As we have shown above, corresponding contribution cancels -1 in formula (15). The possibility of such a cancellation was conjectured by Girvin [17].

The authors are grateful to A. M. Finkelstein, M. P. A. Fisher, S. M. Girvin, C. L. Kane, A. I. Larkin, I. M. Ruzin, and B. I. Shklovskii for helpful discussions. This work was supported by NSF Grants No. DMR-9117341 and No. PHY89-04035, and by Research Fund of the Graduate School of the University of Minnesota.

[a] On leave from the Institute of Solid State Physics, Chernogolovka, Moscow distr., 142432, Russia.

[1] G.-L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum, New York and London, 1992).
[2] Yu. V. Nazarov, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 105 (1989) [JETP Lett. **49**, 126 (1989)].
[3] M. H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990).
[4] S. M. Girvin, L. I. Glazman, M. Jonson, D. R. Penn, and M. D. Stiles, Phys. Rev. Lett. **64**, 3183 (1990).
[5] See, e.g., a review: G. Timp, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).
[6] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).
[7] L. I. Glazman, I. M. Ruzin, and B. I. Shklovskii, Phys. Rev. B **45**, 8454 (1992).
[8] A. A. Odintsov, Phys. Rev. B **45**, 13 717 (1992).
[9] This obvious result is consistent also with theory [6] because the substitution of compressibility for noninteracting electrons $\kappa = m/\pi^2\hbar^2 n$ into formula (3) gives $\alpha = 0$.
[10] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).
[11] J. Sólyom, Adv. Phys. **28**, 201 (1979).
[12] H. J. Lipkin, *Quantum Mechanics* (Elsevier, New York, 1973).
[13] V. J. Emery, in *Highly Conducting One-Dimensional Solids*, edited by J. T. Devreese, R. P. Evrard, and V. E. van Doren (Plenum, New York, 1979).
[14] M. A. Kastner, Rev. Mod. Phys. **64**, 849 (1992).
[15] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990), 2nd ed., Sec. 4.4.
[16] C. L. Kane and M. P. A. Fisher (to be published).
[17] S. M. Girvin (private communication).