Theory of strong inelastic cotunneling

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We develop a theory of the conductance of a quantum dot connected to two leads by single-mode quantum point contacts. If the contacts are in the regime of perfect transmission, the conductance shows no Coulomb blockade oscillations as a function of the gate voltage. In the presence of small reflection in *both* contacts, the conductance develops small Coulomb blockade oscillations. As the temperature of the system is lowered, the amplitude of the oscillations grows, and eventually sharp periodic peaks in conductance are formed. Away from the centers of the peaks the conductance vanishes at low temperatures as T^2, in agreement with the theory of inelastic cotunneling developed for the weak-tunneling case. Conductance near the center of a peak can be studied using an analogy with the multichannel Kondo problem. In the case of symmetric barriers, the peak conductance at $T \to 0$ is of the order of $e^2/\hbar$. In the asymmetric case, the peak conductance vanishes linearly in temperature.

I. INTRODUCTION

Electronic transport in mesoscopic systems is usually affected by the Coulomb interactions between the electrons. The interactions manifest themselves most dramatically in small tunnel junctions. If a small metallic grain is connected to a lead by a tunnel junction, an electron tunneling into such a grain charges it by the elementary charge e and increases the energy of the system by $e^2/2C$. For a small grain, the typical capacitance of the system can be very small, so that the charging energy $e^2/2C$ can significantly exceed the temperature of the system T. In this regime one observes the Coulomb blockade: strong suppression of the tunneling into the grain at low temperatures. Various aspects of the Coulomb blockade have been intensively studied in recent years, both experimentally and theoretically.[1,2]

Recently it has become possible[3] to observe the Coulomb blockade in semiconductor heterostructures. Instead of a metallic grain, in such experiments one creates a small region of the two-dimensional electron gas (2DEG)—a quantum dot—separated from the rest of the 2DEG by potential barriers created by applying negative voltage to the gates, Fig. 1. Experimentally, one measures the linear conductance through the quantum dot as a function of the voltage V_g applied to the central gates. The role of the gate voltage is to change the electrostatic energy of the system,

$$E = \frac{(Q - eN)^2}{2C}, \tag{1}$$

where Q is the charge of the quantum dot, and N is a dimensionless parameter proportional to the gate voltage and the capacitance C_g between the dot and the gate, $N = C_g V_g/e$. If the potential barriers separating the quantum dot from the leads are high, the charge Q is an integer in units of e. Thus, the ground state of the system corresponds to some integer value of charge, and the states with different values of Q are separated from it by the electrostatic gap $\Delta \sim e^2/2C$. It is important to note that at the values of the gate voltage corresponding to half-integer values of N the gap vanishes; i.e., the ground state is degenerate in charge Q. For example, at $N = \frac{1}{2}$ the ground state can have charge either $Q = 0$ or e. Therefore at half-integer N the tunneling of an electron into or out of the quantum dot does not lead to the increase of the electrostatic energy of the system, and the Coulomb blockade is lifted. As a result the conductance of the structure in Fig. 1 shows periodic peaks as a function of N. The Coulomb blockade peaks in linear conductance as a function of the gate voltage are readily observed in the experiments (see, e.g., Ref. 3).

The shapes and heights of the peaks in conductance

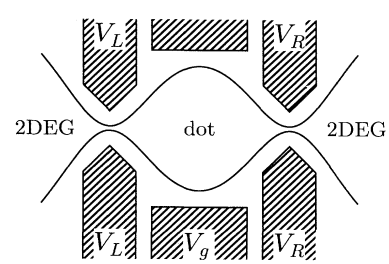

FIG. 1. Schematic view of a quantum dot connected to two bulk 2D electrodes. The dot is formed by applying negative voltage to the gates (shaded). Solid line shows the boundary of the 2D electron gas (2DEG). Electrostatic conditions in the dot are controlled by the voltage applied to the central gates. Voltage $V_{L,R}$ applied to the auxiliary gates controls the transmission probability through the left and right constrictions.

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are usually described within the framework of the orthodox theory of the Coulomb blockade.[1] In this approach the conductance is found from the balance of the rates of tunneling of electrons between the leads and the dot, with the rates being calculated in the lowest-order perturbation theory in the tunneling matrix elements. The resulting linear conductance has the form[4]

$$G = \frac{1}{2}\frac{G_L G_R}{G_L + G_R}\frac{\Delta/T}{\sinh(\Delta/T)}, \quad \Delta = \frac{e^2}{C}\delta N. \tag{2}$$

Here G_L and G_R are the conductances of the left and right barriers, δN is the deviation of N from the nearest half-integer value. Away from the center of the peak, the tunneling of an electron into the quantum dot is suppressed, because only a small fraction of electrons has the energy sufficient to overcome the electrostatic gap Δ. Thus the conductance (2) decays exponentially at $T \to 0$. At very low temperatures, however, another mechanism—usually referred to as *inelastic cotunneling*—dominates the conductance. Instead of a real process of tunneling from a lead to the dot, an electron tunnels from the left lead to a virtual state in the dot, and then another electron tunnels from the dot to the right lead. As a result of such a two-step process, one electron is transferred from the left lead to the right one, and the charge of the quantum dot is unchanged. The corresponding conductance has only a power-law dependence on temperature,[5]

$$G = \frac{\pi\hbar G_L G_R}{3e^2}\left(\frac{T}{\Delta}\right)^2, \quad T \ll \Delta. \tag{3}$$

In the case of weak tunneling, $G_L, G_R \ll e^2/\hbar$, the results (2) and (3) give the full description of the Coulomb blockade peaks in linear conductance. Near the center of the peak the conductance is given by Eq. (2), whereas the tails of the peaks are dominated by the cotunneling contribution (3).

In a number of recent experiments[6–9] the tunneling through quantum dots was studied in the regime where one or more quantum point contacts were tuned to the regime of strong tunneling, corresponding to conductance $G \to e^2/\pi\hbar$. In particular, it was demonstrated by van der Vaart *et al.*[6] that the Coulomb blockade oscillations of linear conductance persist as long as the conductances of both barriers are below the conductance quantum $e^2/\pi\hbar$. Most of the existing theoretical works on the strong-tunneling regime of the Coulomb blockade[10–13] discuss the case of metallic systems, where one can achieve large conductance of a tunnel junction between the grain and a lead by increasing the number of transverse modes while keeping the transmission coefficient $\mathcal{T}$ small. Such theories cannot be applied to the experiments in semiconductor heterostructures, where the point contacts typically allow only a single transverse mode, but the transmission coefficient can be tuned to $\mathcal{T} \to 1$. The theoretical works for this case either do not concentrate on the Coulomb blockade oscillations[14] or study the oscillations of equilibrium properties of the system,[15] rather than its conductance.

In this paper we develop a theory of the Coulomb blockade oscillations of conductance in the regime of strong tunneling. It was shown in Ref. 15 that the average charge of a quantum dot connected to a lead by a single-mode quantum point contact shows small periodic oscillations as a function of the gate voltage as long as the transmission coefficient $\mathcal{T}$ is below unity. Similarly, we will show that the conductance through a quantum dot shows small periodic oscillations at $\mathcal{T} < 1$. One should note, however, that unlike the average charge,[15] the amplitude of the oscillations of conductance depends on temperature and grows as T is lowered. In the limit $T \to 0$, small oscillations develop into sharp periodic peaks, and the off-peak conductance vanishes as T^2, thus reproducing the characteristic quadratic temperature dependence for inelastic cotunneling (3).

In Sec. II we use the analogy[16,17] between the Coulomb blockade and multichannel Kondo problem to discuss the peak value of conductance at $T \to 0$. This discussion is done for the weak-tunneling case, but the conclusion that the peak conductance is of the order of $e^2/\hbar$ is valid for the strong tunneling as well. The theory of conductance oscillations in the strong-tunneling regime is developed in Secs. III and IV. In Sec. V we discuss the scaling properties of our problem. In particular, we show that the quadratic temperature dependence of the off-peak conductance at $T \to 0$ is universal and holds for arbitrary transmission coefficients.

II. CONDUCTANCE PEAKS IN THE WEAK-TUNNELING CASE

Let us first consider the weak-tunneling case. The linear conductance is given by Eqs. (2) and (3). One should note that in the derivation of Eqs. (2) and (3) only the lowest-order terms of the perturbation theory in tunneling amplitudes were taken into account. This is usually justified at $G_L, G_R \ll e^2/\hbar$. However, it was shown in Ref. 16 that near a half-integer N and at low temperatures the higher-order corrections lead to large logarithmic renormalizations of the tunneling amplitudes. The nature of these renormalizations can be understood in terms of the analogy[17] between the Coulomb blockade and the Kondo problem. We will now discuss this analogy and correct the results (2) and (3) to take into account the renormalizations of the tunneling amplitudes.

For simplicity we start with the quantum dot weakly coupled to a *single* lead. Such a system can be described by the following Hamiltonian:

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \sum_p \epsilon_p a_p^\dagger a_p + E_C(\hat{n} - N)^2 + \sum_{kp}(v_t a_k^\dagger a_p + v_t^* a_p^\dagger a_k). \tag{4}$$

Here a_k and a_p are the annihilation operators for the electrons in the lead and in the dot, respectively, and E_C is the characteristic charging energy, $E_C = e^2/2C$; coupling between the dot and the lead is described by a

tunneling Hamiltonian with the matrix element v_t. Finally, $\hat{n}$ is the operator of the number of electrons in the dot, counted from that at the ground state of the system without tunneling, $\hat{n} = \sum(a_p^\dagger a_p - \langle a_p^\dagger a_p\rangle_0)$.

Due to the last term in Eq. (4) electrons can tunnel from the lead to a virtual state in the dot. The energy of such a virtual state is of the order of E_C, and to complete this scattering process an electron has to tunnel from the dot to the lead. It is instructive to calculate the second-order amplitude of scattering between two states in the lead, k and k', near the Fermi level:

$$T_{k\to k'} = \sum_p \left[\frac{|v_t|^2\theta(\epsilon_p)}{E_C N^2 - E_C(1-N)^2 - \epsilon_p} - \frac{|v_t|^2\theta(-\epsilon_p)}{E_C N^2 - E_C(1+N)^2 + \epsilon_p}\right]. \tag{5}$$

Here the second term corresponds to the process in which first an electron from the filled state p in the dot tunnels to the state k' in the lead, and then the electron k fills the hole in the dot. Clearly, the total amplitude diverges logarithmically at $N \to \pm\frac{1}{2}$,

$$T_{k\to k'} = -\nu_d|v_t|^2 \ln\frac{1+2N}{1-2N}, \tag{6}$$

where ν_d is the density of states in the dot. The logarithmic singularity at $N \to \frac{1}{2}$ originates from the degeneracy of the electrostatic energy for states with $n = 0$ and $n = 1$ extra electrons in the dot.

Let us now concentrate on the case of the dimensionless gate voltage N very close to $\frac{1}{2}$. In this case the logarithmic renormalizations are large, and to find the scaling properties of the scattering amplitudes, we can restrict our consideration to the energy scales smaller than E_C. Thus when constructing the perturbation series for a tunneling amplitude we will neglect all the processes in which the system visits virtual states with the charge n different from 0 or 1. Another important property of the Hamiltonian (4) is that the perturbation—the tunneling term—always changes the charge of the dot by 1. Suppose that at the first step of a high-order tunneling process an electron tunnels from the lead to the dot thus changing n from 0 to 1. Then at the next step an electron will have to tunnel from the dot to the lead (changing $n = 1 \to 0$), because otherwise a virtual state with $n = 2$ and correspondingly large energy $\sim E_C$ would have been created. At the third step an electron tunnels from the lead to the dot, changing $n = 0 \to 1$, and so on.

One can easily see that exactly the same perturbation series for scattering amplitudes is obtained from the Hamiltonian of the anisotropic Kondo model,

$$H = \sum_{k\alpha} \epsilon_k a_{k\alpha}^\dagger a_{k\alpha} + hS^z + J_\perp \sum_{kk'\alpha\alpha'} (\sigma_{\alpha\alpha'}^+ S^- + \sigma_{\alpha\alpha'}^- S^+)\, a_{k\alpha}^\dagger a_{k'\alpha'}, \tag{7}$$

where $\sigma^\pm$ are the linear combinations of Pauli matrices, $\sigma^\pm = \sigma^x \pm \sigma^y$, and $\boldsymbol{S}$ is the impurity spin, $S^\pm = S^x \pm S^y$. A formal discussion of this mapping can be found in Ref. 17; here we only mention the relations between the parameters of the Hamiltonians (4) and (7). The values of the electron spin $\alpha = \uparrow$ and $\downarrow$ correspond to the electron being in the lead and the dot, respectively. Similarly, the values of the impurity spin $S^z = \downarrow$ and $\uparrow$ correspond to the states with the number of particles in the dot $n = 0$ and 1. Magnetic field h is identified with the energy splitting of the $n = 0$ and $n = 1$ states, $h = 2E_C(\frac{1}{2} - N)$. Finally, the tunneling matrix element v_t is mapped to the exchange constant $J_\perp$.

It is well known that at low temperatures and magnetic fields the exchange constant J of the Kondo model renormalizes logarithmically. As a result J grows, and the system approaches the strong-coupling fixed point. In the context of the Coulomb blockade problem, this means that at $N \to \frac{1}{2}$ and $T \to 0$ the tunneling amplitudes grow until the conductance of the barrier becomes of order $e^2/\hbar$.

Using the analogy with the Kondo problem, one can actually find the renormalizations of the tunneling amplitudes. In the leading logarithm approximation one can use the poor man's scaling technique.[18] The result[16] can be written down in terms of the renormalization of the transmission coefficient $\mathcal{T}$ of the barrier between the lead and the dot,

$$\mathcal{T} = \frac{\mathcal{T}_0}{\cos^2\left(\frac{1}{\pi}\sqrt{\mathcal{T}_0}\xi\right)}, \quad \xi = \ln\frac{E_C}{\max\{T,\Delta\}}. \tag{8}$$

Here $\mathcal{T}_0$ is the bare value of the transmission coefficient, and $\Delta = E_C|1-2N|$. When $\mathcal{T}$ becomes of the order of unity, the leading logarithm approximation fails. Thus the result (8) is valid as long as the renormalized transmission coefficient is small, $\mathcal{T} \ll 1$.

In this regime one can still use the weak-tunneling results (2) and (3) for the conductance through a dot connected to two leads, taking into account the renormalizations of the conductances of the barriers. This can be done by noting the relation between the conductance of a barrier G_b and its transmission coefficient,

$$G_b = \frac{e^2}{2\pi\hbar} \times \begin{cases} \mathcal{T}, & \text{without spins,} \\ 2\mathcal{T}, & \text{with spins.} \end{cases} \tag{9}$$

It is worth mentioning that within the leading logarithm approximation the transmission coefficients of two barriers renormalize independently, so one can use Eq. (8) for both left and right barriers.

Let us find the temperature dependence of the peak value of the conductance in the case of symmetric barriers, i.e., assume $N = \frac{1}{2}$ and $G_L = G_R = G_b = (e^2/\pi\hbar)\mathcal{T}$. According to Eq. (2), the peak conductance is $\frac{1}{4}G_b$. Taking into account the renormalization (8) of the transmission coefficient, we find

$$G = \frac{e^2}{4\pi\hbar}\frac{\mathcal{T}_0}{\cos^2\left[\frac{1}{\pi}\sqrt{\mathcal{T}_0}\ln(E_C/T)\right]}. \tag{10}$$

As expected, when the temperature is lowered, the conductance grows. This result is valid only at temperatures exceeding the characteristic Kondo temperature of this

problem, $T_K \simeq E_C \exp(-\pi^2/2\sqrt{\mathcal{T}_0})$. At too low temperatures, $T \lesssim T_K$, the conductance is of the order of $e^2/\hbar$, transmission coefficient $\mathcal{T} \sim 1$, and the result of the leading logarithm approximation (10) is no longer applicable.

It is also interesting to study the tails of the conductance peaks. If the gate voltage N is not precisely half-integer, then at low temperatures, $T \ll \Delta = 2E_C|N-\frac{1}{2}|$, one can still use the inelastic cotunneling result (3) with the values of conductances of the barriers renormalized according to Eqs. (8) and (9). The resulting conductance has the form:

$$G = \frac{e^2\mathcal{T}_L\mathcal{T}_R}{3\pi\hbar}\frac{T^2}{\Delta^2\cos^2\left[\frac{1}{\pi}\sqrt{\mathcal{T}_L}\ln\frac{E_C}{\Delta}\right]\cos^2\left[\frac{1}{\pi}\sqrt{\mathcal{T}_R}\ln\frac{E_C}{\Delta}\right]}. \tag{11}$$

Similarly to Eq. (10), the result (11) is valid only as long as $\mathcal{T} \ll 1$. In this case the condition allows only the values of the gate voltage not too close to the resonance, $\Delta \gg T_K$.

As we have seen, if one starts with the weak-tunneling case, in the vicinity of the resonances in $G(V_g)$ the tunneling amplitude grows, and the system approaches the strong-coupling regime. Therefore one can use the results of the weak-tunneling approach to find the universal features of the peaks in conductance, which should also persist in the strong-tunneling case. Such features are (i) the $T \to 0$ value of the peak conductance (10) for symmetric barriers is of the order of $e^2/\hbar$, and (ii) away from the centers of peaks the temperature dependence of the conductance at $T \to 0$ is $G \propto T^2$. It is worth noting that unlike the temperature dependence of the cotunneling conductance (3), its $1/\Delta^2$ dependence of the gate voltage is not universal; see Eq. (11).

III. STRONG COTUNNELING OF SPINLESS ELECTRONS

We will start our treatment of the strong-tunneling case with a simplified model of spinless electrons. As we will see, in this case a complete solution of the problem is possible. Experimentally, the spinless case can be achieved by applying an in-plane magnetic field, sufficiently strong to polarize the electron gas in the constrictions.

A. Theoretical model

We assume that each of the quantum point contacts in Fig. 1 allows the transmission of electrons in only one mode. The transmission coefficients of the two contacts $\mathcal{T}_L$ and $\mathcal{T}_R$ are assumed to be close to unity. This enables us to start with the Hamiltonian corresponding to perfect transmission, $\mathcal{T}_L = \mathcal{T}_R = 1$, and then construct the perturbation theory in small reflection amplitudes, r_L and r_R. Following Ref. 14, we will describe each of the quantum point contacts by a system of one-dimensional (1D) electrons. The formal derivation of this model[15] is based on the fact that if the boundaries of the channel are very smooth, the transport though it can be described in terms of adiabatic wave functions[19] having essentially a 1D form.

Another simplification of the problem is due to the fact that the typical energies involved in the Coulomb blockade phenomenon are of the order of the charging energy E_C, which is much smaller than the Fermi energy. This allows us to linearize the spectra of the 1D electrons near the Fermi points and to formulate the model in terms of the left- and right-moving fermions:

$$H_0 = i\hbar v_F \int [\psi^\dagger_{L1}(x)\nabla\psi_{L1}(x) - \psi^\dagger_{L2}(x)\nabla\psi_{L2}(x) + \psi^\dagger_{R1}(x)\nabla\psi_{R1}(x) - \psi^\dagger_{R2}(x)\nabla\psi_{R2}(x)]dx. \tag{12}$$

Here ψ_{L1} and ψ_{L2} are the annihilation operators for the electrons of the left contact moving to the left and right, respectively; ψ_{R1} and ψ_{R2} describe the left- and right-moving electrons of the right contact; $\nabla \equiv \frac{\partial}{\partial x}$. We associate the centers of the constrictions with points $x = 0$, meaning that electrons in the dot are described by $\psi_{L1(2)}$ at $x > 0$ and by $\psi_{R1(2)}$ at $x < 0$. Thus the charging energy can be presented as

$$H_C = E_C(\hat{n} - N)^2, \tag{13}$$

where

$$\hat{n} = \int_0^\infty \sum_{j=1,2}\left[:\psi^\dagger_{Lj}(x)\psi_{Lj}(x): + :\psi^\dagger_{Rj}(-x)\psi_{Rj}(-x):\right]dx.$$

Here $:\hat{X}:$ denotes the normal-ordered operator $\hat{X}$. Finally, the presence of weak backscattering in the contacts, described by small reflection amplitudes $|r_L|, |r_R| \ll 1$, gives rise to the term mixing the left- and right-moving particles,

$$H' = \hbar v_F[|r_L|\psi^\dagger_{L1}(0)\psi_{L2}(0) + |r_R|\psi^\dagger_{R1}(0)\psi_{R2}(0) + \text{H.c.}]. \tag{14}$$

The Hamiltonian $H = H_0 + H_C + H'$ gives the complete description of the transport through a quantum dot connected to two leads. In our model the two contacts are described by two independent 1D systems. Thus we neglect the possibility of coherent transport of electrons from one quantum point contact to the other. Such processes lead to the additional mechanism of transport through the dot called *elastic cotunneling.* It is known,[5] however, that at temperatures exceeding the level spacing ε in the dot the elastic cotunneling contribution to the total conductance is much smaller than that of inelastic cotunneling. Thus we will concentrate on the case $T \gg \varepsilon$, which is easy to satisfy in the experiments, and in the following calculations we will assume $\varepsilon = 0$. At $T \lesssim \varepsilon$ the conductance becomes temperature independent and can be estimated by replacing T by ε in our results.

The solution of the problem with the Hamiltonian given by Eqs. (12)–(14) is not trivial because the interaction term H_C is not quadratic in fermion operators.

To overcome this difficulty, we will use the bosonization technique,[20] which enables one to treat the interaction in 1D systems exactly. Following Ref. 20, we present the fermion operators as

$$\begin{aligned}\psi_{L1}(x) &= \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2}\eta_{L1}e^{i\varphi_{L1}(x)},\\ \psi_{L2}(x) &= \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2}\eta_{L2}e^{i\varphi_{L2}(x)}.\end{aligned} \tag{15}$$

Here D is the bandwidth, and the two independent bosonic fields φ_{L1} and φ_{L2} satisfy the following commutation relations:

$$\begin{aligned}[\varphi_{L1}(x),\varphi_{L1}(y)] &= -[\varphi_{L2}(x),\varphi_{L2}(y)]\\ &= -i\pi\,\mathrm{sgn}(x-y).\end{aligned} \tag{16}$$

Given (16), the operators (15) satisfy usual anticommutation relations. To ensure proper anticommutation relations for fermions at different branches, we have added the local Majorana fermions η_{L1} and η_{L2}.

The electrons of the right contact are bosonized in a similar way. One can then rewrite the Hamiltonian (12)–(14) in terms of φ_L and φ_R. It will be more convenient, however, to present the Hamiltonian in terms of slightly different bosonic fields:

$$\phi_L = \frac{1}{2}(\varphi_{L2}-\varphi_{L1}), \quad \Pi_L = -\frac{1}{2\pi}\nabla(\varphi_{L1}+\varphi_{L2}). \tag{17}$$

The new variables satisfy the commutation relations typical for the displacement of a 1D elastic medium and its momentum density,

$$[\phi_L(x),\Pi_L(y)] = i\delta(x-y). \tag{18}$$

The form of the free-electron Hamiltonian (12) supports this observation,

$$\begin{aligned}H_0 = \frac{\hbar v_F}{2}\int\Big\{&\frac{1}{\pi}[\nabla\phi_L(x)]^2+\pi\Pi_L^2(x)\\ &+\frac{1}{\pi}[\nabla\phi_R(x)]^2+\pi\Pi_R^2(x)\Big\}\,dx.\end{aligned} \tag{19}$$

This qualitative interpretation allows one to predict the correct bosonized form of the charging energy (13). One should expect that the charge of the dot is proportional to the difference of the charge $Q_L \sim e\phi_L(0)$ brought into the dot through the left contact and the charge $Q_R \sim e\phi_R(0)$ carried away through the right contact. Explicit calculation gives

$$H_C = \frac{E_C}{\pi^2}\left[\phi_R(0)-\phi_L(0)-\pi N\right]^2. \tag{20}$$

It is important to stress that the interaction term (20) is now quadratic in ϕ_L and ϕ_R. Therefore the bosonization approach enables one to treat the charging energy exactly.

Finally, the backscattering term (14) takes the form

$$H' = \frac{D|r_L|}{\pi}\cos[2\phi_L(0)] + \frac{D|r_R|}{\pi}\cos[2\phi_R(0)]. \tag{21}$$

In Eq. (21) we neglected the products of Majorana fermions $\eta_{L1}\eta_{L2}$ and $\eta_{R1}\eta_{R2}$, as they commute with each other. Unlike the first two terms, H_0 and H_C, the backscattering term is not quadratic in bosonic variables and cannot be treated exactly. Below we assume weak backscattering and develop the perturbation theory in small parameters $|r_L|$ and $|r_R|$.

B. Perturbation theory for the conductance

In this paper we are only interested in the linear conductance through the quantum dot. To find it, we will use a Kubo formula, which in the zero-frequency limit can be written as

$$G = \frac{i}{2\hbar}\int_{-\infty}^{\infty} t\langle[I(t),I(0)]\rangle dt, \tag{22}$$

where I is the operator of current through the dot.

We define I as the average of the current I_L flowing into the dot through the left contact and the current I_R flowing out of the dot through the right contact. Clearly, the current I_L is proportional to the time derivative of the displacement of the effective elastic medium describing the left contact, $I_L \sim e\dot{\phi}_L$, and, similarly, $I_R \sim e\dot{\phi}_R$. From the equations of motion corresponding to the Hamiltonian $H_0 + H_C + H'$, one concludes that $I \sim ev_F(\Pi_L + \Pi_R)$. A careful calculation gives

$$I = \frac{1}{2}ev_F\,(\Pi_L+\Pi_R)|_{x=0}\,. \tag{23}$$

As the first step, we will find the conductance in the absence of the barriers. Since the current operator (23) commutes with H_C, one should expect the conductance to be insensitive to the charging energy. Clearly, at $E_C = 0$ the system is equivalent to two resistances $2\pi\hbar/e^2$ of the two quantum point contacts, connected in series. One then expects the conductance to be

$$G_0 = \frac{e^2}{4\pi\hbar}. \tag{24}$$

This result is easily derived from the Kubo formula (22), because with $H' = 0$ the current-current correlator must be found for a quadratic Hamiltonian. As a result, one finds $\langle[I(t),I(0)]\rangle = i(e^2/2\pi)\delta'(t)$, and after the substitution into (22) one reproduces (24).

The result (24) is independent of temperature. Since we used the Hamiltonian (12) with linearized spectra of electrons, the temperature was assumed to be much smaller than the Fermi energy, but could still be of the order of the charging energy. In the presence of the barriers the conductance must acquire some temperature dependence, which is obvious from the limit of very high barriers (3). Indeed, the lowest nonvanishing (second-order) correction in r_L and r_R to the conductance is already temperature dependent. At low temperatures $T \ll E_C$, we get

$$G = \frac{e^2}{4\pi\hbar}\left(1-\frac{\pi\Gamma_0(N)}{4T}\right), \tag{25}$$

where Γ_0 is defined as follows:

$$\Gamma_0(N) = \frac{2\gamma E_C}{\pi^2}\left[|r_L|^2 + |r_R|^2 + 2|r_L||r_R|\cos(2\pi N)\right]. \tag{26}$$

Here $\gamma = e^{\mathbf{C}}$, with $\mathbf{C} \approx 0.5772,\ldots$ being the Euler's constant. In Eq. (25) we have neglected the terms small in T/E_C. The result (25) and (26) is analogous to that for the conductance of a 1D wire with two defects[21,22] and can be obtained in a similar way, see Appendix A 1.

As expected, the presence of weak scattering in the contacts gives rise to a small periodic correction to the conductance. The same behavior was predicted[15] for the average charge of the dot. However, unlike the average charge, the correction to conductance strongly depends on temperature. As the temperature is lowered, the conductance decreases. The only case when conductance is temperature independent is when the barriers are symmetric, $|r_L| = |r_R|$, and the dimensionless gate voltage N is half-integer, which corresponds to the center of a peak in the high-barrier case. In this regime we expect the conductance to be of the order of $e^2/\hbar$, cf. Sec. II, which is confirmed by the perturbation theory (25) and (26).

C. Nonperturbative calculation

At low temperatures $T \lesssim \Gamma_0$ the second-order correction becomes large, and the perturbation theory fails. To find the conductance in this case, one has to sum up an infinite number of terms of the perturbation theory. To perform such a summation, we will use the technique developed in Ref. 15.

At the first step we perform a linear transformation of the bosonic fields,

$$\phi_L = \frac{\phi_I - \phi_C}{\sqrt{2}}, \quad \phi_R = \frac{\phi_I + \phi_C}{\sqrt{2}}, \tag{27}$$

and similarly for Π_L and Π_R. The advantage of using these new variables is that the Coulomb energy (20) is now expressed in terms of the charging mode ϕ_C only,

$$H_C = \frac{2E_C}{\pi^2}\left[\phi_C(0) - \frac{\pi N}{\sqrt{2}}\right]^2. \tag{28}$$

The charging energy (28) suppresses the fluctuations of $\phi_C(0)$ at energy scales below E_C. Thus at $T \ll E_C$ one can integrate out the fluctuations of the charging mode by replacing the scattering term (21) with its value averaged over the fluctuations of ϕ_C. The resulting Hamiltonian is expressed in terms of the field ϕ_I only and has the form

$$H_0 = \frac{\hbar v_F}{2}\int\left\{\frac{1}{\pi}[\nabla\phi_I(x)]^2 + \pi\Pi_I^2(x)\right\}dx, \tag{29}$$

$$H' = \left(\frac{\gamma E_C D}{2\pi^3}\right)^{1/2}\left[re^{i\sqrt{2}\phi_I(0)} + r^* e^{-i\sqrt{2}\phi_I(0)}\right], \tag{30}$$

where the dependence on the gate voltage is incorporated into the complex parameter $r = |r_L|e^{-i\pi N} + |r_R|e^{i\pi N}$.

The Hamiltonian (29) and (30) can be transformed to that of an impurity in a $g = \frac{1}{2}$ Luttinger liquid[23] by a simple transformation $\phi = \sqrt{2}\phi_I$ and $\Pi = \Pi_I/\sqrt{2}$. The latter can be solved exactly[23] using the technique developed in Ref. 24. For our purposes it will be more convenient to use an alternative exact solution.[15] The idea is to interpret $e^{i\sqrt{2}\phi_I}$ as a bosonized form of some fermion annihilation operator.[25] This can be done by rewriting the Hamiltonian in terms of two decoupled chiral bosonic fields defined as

$$\varphi_\pm(x) = \frac{\phi_I(x) \pm \phi_I(-x)}{\sqrt{2}} + \pi\int_0^x \frac{\Pi_I(x') \pm \Pi_I(-x')}{\sqrt{2}}dx'. \tag{31}$$

In these variables the Hamiltonian takes the form

$$H_0 = \frac{\hbar v_F}{4\pi}\int\left\{[\nabla\varphi_+(x)]^2 + [\nabla\varphi_-(x)]^2\right\}dx, \tag{32}$$

$$H' = \left(\frac{\gamma E_C D}{2\pi^3}\right)^{1/2}\left[re^{i\varphi_+(0)} + r^* e^{-i\varphi_+(0)}\right]. \tag{33}$$

One can easily check that the fields φ_+ and φ_- commute. Thus the φ_- part of the Hamiltonian completely decouples. Since the field $\varphi_+(x)$ satisfies the same commutation relations (16) as φ_{L1}, one can in complete analogy with Eq. (15) interpret $e^{i\varphi_+}$ as a fermion operator,[26]

$$e^{i\varphi_+(x)} = \left(\frac{2\pi\hbar v_F}{D}\right)^{1/2}\eta_+\psi_+(x). \tag{34}$$

The resulting Hamiltonian is quadratic in fermion operators,

$$H = i\hbar v_F\int\psi_+^\dagger(x)\nabla\psi_+(x)dx + \left(\frac{\gamma\hbar v_F E_C}{\pi^2}\right)^{1/2}\left[r\eta_+\psi_+(0) + r^*\psi_+^\dagger(0)\eta_+\right], \tag{35}$$

and can be easily diagonalized.[15] This greatly simplifies the calculation of the conductance. One can again use the Kubo formula (22), with the properly transformed current operator (23). In the new fermionic variables the current operator has the form

$$I = -ev_F :\psi_+^\dagger\psi_+:\Big|_{x=0}. \tag{36}$$

The actual calculation of the linear conductance is rather long, but straightforward. First we find the current-current correlation function to be

$$\langle[I(t), I(0)]\rangle = i\frac{e^2}{2\pi}\delta'(t) - i\frac{e^2}{2\pi^2\hbar^2} \times\int\int\frac{\Gamma_0^2\sin[(\epsilon+\epsilon')t/\hbar]d\epsilon d\epsilon'}{(\epsilon^2+\Gamma_0^2)(e^{\epsilon/T}+1)(e^{\epsilon'/T}+1)}. \tag{37}$$

The substitution of Eq. (37) into the Kubo formula (22)

gives the following expression for the linear conductance:

$$G = \frac{e^2}{4\pi\hbar}\left[1 - \int_{-\infty}^{\infty} dE \frac{1}{4T\cosh^2\frac{E}{2T}} \frac{\Gamma_0^2(N)}{E^2 + \Gamma_0^2(N)}\right]. \quad (38)$$

Equation (38) is the central result of this section. It is valid for any temperature below E_C, and, unlike the perturbative result (25), it is not restricted to only sufficiently high temperatures $T \gg \Gamma_0$. Of course, at $T \gg \Gamma_0$ the result (25) is reproduced.

In the opposite limit, $T \ll \Gamma_0$, the conductance is quadratic in temperature,

$$G = \frac{\pi e^2}{12\hbar}\left[\frac{T}{\Gamma_0(N)}\right]^2. \quad (39)$$

It is important to note that the same temperature dependence was found for the inelastic cotunneling (3) in the weak-tunneling regime. As expected, the quadratic temperature dependence is a universal property of inelastic cotunneling and is not limited to the weak-tunneling limit; cf. Eq. (11).

In the special case of symmetric barriers, $|r_L| = |r_R| = r_0$, the low-temperature limit (39) is never achieved on resonance. Indeed, we have $\Gamma_0 = (8\gamma E_C/\pi^2) r_0^2 \cos^2 \pi N$; i.e., Γ_0 vanishes at half-integer values of N. Therefore in the low-temperature limit the resonant value of conductance is $e^2/4\pi\hbar$, whereas away from the resonance $\Gamma_0 \neq 0$, and $G \propto T^2$. It is clear then that at low temperatures the weak Coulomb blockade oscillation in conductance (25) transforms into sharp periodic peaks, Fig. 2. Unlike the weak-tunneling case (3), the tails of the peak at $N = \frac{1}{2}$ are not Lorentzian: $G \propto 1/(N - \frac{1}{2})^4$.

Finally, it is worth noting that the conductance (38) coincides with that of a weak impurity in the $g = \frac{1}{2}$ Luttinger liquid.[23] This is not surprising, considering the aforementioned identity of the Hamiltonian (29) and (30) with the one for the quantum impurity problem.[23] In the latter problem the condition $g = \frac{1}{2}$ requires special fine tuning of the strength of interaction between electrons. In our case the condition $g = \frac{1}{2}$ is satisfied automatically when we integrate out one of the two bosonic modes.[21]

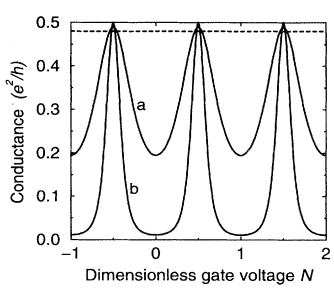

FIG. 2. Conductance (38) as a function of the dimensionless gate voltage N for the symmetric case, $|r_L| = |r_R| = 0.2$. The two curves are calculated for $E_C/T =$ (a) 25 and (b) 200. In this calculation we included the correction proportional to T/E_C in Eq. (A9). The dashed line represents the conductance $G = G_L G_R/(G_L + G_R)$ in the high-temperature limit, $T \gg E_C$.

IV. COTUNNELING OF ELECTRONS WITH SPINS

In the previous section we considered the case of spinless electrons, which can be realized by applying a strong in-plane magnetic field. In the absence of the field, each of the contacts has two identical modes, corresponding to two possible values of electron spin. The number of modes is an important parameter in our problem. This can be easily seen from the analogy to the Kondo problem discussed in Sec. II. In the weak-tunneling case, the model of spinless electrons maps to the two-channel Kondo problem, because the fictitious spin of the impurity—i.e., the charge of the dot—can be changed by tunneling of an electron in two independent channels, corresponding to the left and right contacts. In the presence of real spins of electrons, the number of channels in the effective Kondo problem is 4. It is well known that the number of channels in the Kondo problem strongly affects its properties.[27] Thus one should expect that the presence of spins should have a dramatic effect upon the transport through a quantum dot.

A. Strong-tunneling case

We will start with the discussion of the strong-tunneling limit, $\mathcal{T}_L, \mathcal{T}_R \to 1$. To find the conductance, one can use the same technique as in Sec. III. The Hamiltonian of the problem can be written in the bosonized form, analogous to (19)–(21),

$$H_0 = \frac{\hbar v_F}{2}\sum_\alpha \int \left\{\frac{1}{\pi}[\nabla\phi_{L\alpha}(x)]^2 + \pi\Pi_{L\alpha}^2(x) + \frac{1}{\pi}[\nabla\phi_{R\alpha}(x)]^2 + \pi\Pi_{R\alpha}^2(x)\right\}dx, \quad (40)$$

$$H_C = \frac{E_C}{\pi^2}\left\{\sum_\alpha[\phi_{R\alpha}(0) - \phi_{L\alpha}(0)] - \pi N\right\}^2, \quad (41)$$

$$H' = \frac{D}{\pi}\sum_\alpha\{|r_L|\cos[2\phi_{L\alpha}(0)] + |r_R|\cos[2\phi_{R\alpha}(0)]\}. \quad (42)$$

Here the fields with $\alpha = \uparrow$ and $\downarrow$ describe the electrons with spins up and down, respectively.

One can again find the linear conductance using the Kubo formula (22), but the current operator (23) must be modified to account for the spins,

$$I = \frac{1}{2} e v_F \sum_{\alpha=\uparrow,\downarrow} (\Pi_{L\alpha} + \Pi_{R\alpha})\Big|_{x=0}. \quad (43)$$

In the absence of barriers, $|r_L| = |r_R| = 0$, the calculation is trivial, as the Hamiltonian (40) and (41) is

quadratic. Due to the doubling of the number of modes in each contact, in the presence of spins the conductance doubles compared to Eq. (24),

$$G_0 = \frac{e^2}{2\pi\hbar}. \tag{44}$$

Weak backscattering, $|r_L|, |r_R| \ll 1$, can be treated in perturbation theory in H'; see Appendix A 2. Up to the second order we get

$$G = \frac{e^2}{2\pi\hbar}\left[1 - \frac{2\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\sqrt{\frac{\gamma E_C}{\pi T}}\left(|r_L|^2 + |r_R|^2\right)\right]. \tag{45}$$

It is instructive to compare this result with its analog for the spinless case, namely, Eq. (25). In both cases the small correction to the conductance has the tendency to grow as the temperature is decreased. An important difference is that the second-order correction in (45) does not depend on the gate voltage N, and as a result it grows even at half-integer N, i.e., on resonance. Thus, in contrast to the spinless case, one should expect the resonance value to be smaller than the unperturbed conductance $G_0 = e^2/2\pi\hbar$.

This difference has a natural interpretation in terms of the analogy with the Kondo model. As we already mentioned, the spinless case corresponds to the two-channel Kondo model. In the two-channel case the strong-coupling fixed point is stable at zero magnetic field, or, in terms of the Coulomb blockade problem, at half-integer N. Therefore, precisely on resonance a weak reflection must be an irrelevant perturbation, and the peak value of the conductance should be given by Eq. (24). On the contrary, the four-channel Kondo problem corresponding to the case with spins has a stable fixed point at an intermediate value of coupling, and both weak- and strong-coupling fixed points are unstable.[27] Thus even on resonance the conductance is not given by its value in the strong-tunneling limit (44), as confirmed by the perturbation theory (45).

B. Conductance in the case of asymmetric barriers

Unfortunately, it is not easy to calculate the conductance near the intermediate-coupling fixed point. Some conclusions about it can be drawn on the basis of the scaling approach, Sec. V. One should note, however, that this fixed point is stable only if the coupling in all four channels is identical. In the case of the Coulomb blockade problem, the coupling in the pairs of channels corresponding to different spin orientations in the same contact must be identical (in the absence of magnetic field). On the other hand, the coupling constants for the modes in different contacts are determined by the corresponding conductances and are not necessarily equal. In fact, in the structure shown in Fig. 1, the two conductances are controlled separately by the gate voltages V_L and V_R. Therefore in experiments the coupling constants corresponding to the left and right modes are not equal, unless a special attempt to make them equal is made.

In this section we study the limit of very asymmetric barriers, namely, we assume that one of the barriers is very high, whereas the other one is low. In terms of the transmission coefficients of the barriers this limit corresponds to $\mathcal{T}_L \ll 1$ and $1 - \mathcal{T}_R \ll 1$. Such a regime can be easily realized in an experiment by proper tuning of the gate voltages V_L and V_R. In addition, we will see in Sec. V that at $T \to 0$ arbitrarily weak asymmetry grows and the system scales towards the strongly asymmetric limit. Therefore at low temperatures the results of this section can be applied to any asymmetric system.

In Sec. II we described the contact in the regime of weak tunneling by the tunnel Hamiltonian (4) written in terms of the original electrons. On the other hand, in Sec. III it was more convenient to use the bosonized form of the Hamiltonian for the electron gas in the contact with transmission coefficient close to unity. Therefore when considering the asymmetric case $\mathcal{T}_L \to 0$ and $\mathcal{T}_R \to 1$ we shall bosonize only the electrons in the right contact. Then the unperturbed fixed-point Hamiltonian $H_0 + H_C$ is given by

$$H_0 = \sum_{k\alpha}\epsilon_k a^\dagger_{k\alpha}a_{k\alpha} + \sum_{p\alpha}\epsilon_p a^\dagger_{p\alpha}a_{p\alpha} + \frac{\hbar v_F}{2}\sum_\alpha\int\left\{\frac{1}{\pi}[\nabla\phi_{R\alpha}(x)]^2 + \pi\Pi^2_{R\alpha}(x)\right\}dx, \tag{46}$$

$$H_C = E_C\left[\hat{n} + \frac{1}{\pi}\sum_\alpha\phi_{R\alpha}(0) - N\right]^2. \tag{47}$$

Here the operators $a_{k\alpha}$ and $a_{p\alpha}$ correspond to the left-contact electrons in the lead and in the dot, respectively; the bosonic fields $\phi_{R\alpha}$ and $\Pi_{R\alpha}$ model the electrons in the right contact. An integer-valued operator $\hat{n}$ describes the number of electrons transferred to the dot through the left barrier.

The perturbation consists of two parts: weak tunneling through the high barrier in the left contact, and weak scattering on the small barrier in the right contact,

$$H'_L = \sum_{kp\alpha}(v_t a^\dagger_{k\alpha}a_{p\alpha}F + v_t^* a^\dagger_{p\alpha}a_{k\alpha}F^\dagger), \tag{48}$$

$$H'_R = \frac{1}{\pi}D|r_R|\sum_\alpha\cos[2\phi_{R\alpha}(0)]. \tag{49}$$

We have chosen not to write the operator $\hat{n}$ of the number of electrons transferred into the dot through the barrier in terms of the fermionic operators as $\hat{n} = \sum a^\dagger_{p\alpha}a_{p\alpha}$, but to treat it as an independent variable. Thus the tunneling Hamiltonian (48) acquires the charge-lowering and raising operators F and $F^\dagger$ defined as

$$[F, \hat{n}] = F. \tag{50}$$

One possible representation[28] of these operators is given by $F = e^{ix}$ and $\hat{n} = i\frac{\partial}{\partial x}$, where x has the meaning of a phase operator conjugated to $\hat{n}$.

Our goal in this section is to find the conductance of the system, assuming that the perturbations H'_L and H'_R are small. At the first step we will utilize the smallness of the amplitude v_t of tunneling through the left barrier,

and find the expression for the current in the second order in v_t. The current can be defined as the time derivative of the number of electrons that have tunneled through the barrier, $I = -e\frac{\partial \hat{n}}{\partial t} = \frac{ie}{\hbar}[\hat{n}, H]$. Using the commutation relation (50), we get

$$I = i\frac{e}{\hbar}\sum_{kp\alpha}\left(v_t^* a_{p\alpha}^\dagger a_{k\alpha} F^\dagger - v_t a_{k\alpha}^\dagger a_{p\alpha} F\right). \tag{51}$$

To find the conductance through the dot, we will substitute the above current operator into the Kubo formula (22). This requires the calculation of the correlator $\langle [I(t), I(0)]\rangle$. Since $I \propto |v_t|$, up to the second order in tunneling matrix elements we can average $[I(t), I(0)]$ over the equilibrium thermal distribution of the system without tunneling, $H_L' = 0$. Then the averaging over the fermionic degrees of freedom is straightforward, and we get

$$G = -iG_L\frac{\pi T^2}{\hbar^2}\int_{-\infty}^{\infty}\frac{tK(t)dt}{\sinh^2[\pi T(t - i\delta)/\hbar]}. \tag{52}$$

Here we have introduced the conductance of the left tunnel barrier, $G_L = (4\pi e^2/\hbar)\sum_{kp}|v_t|^2\delta(\epsilon_k)\delta(\epsilon_p)$, and the correlator

$$K(t) = \langle F(t)F^\dagger(0)\rangle = \langle F^\dagger(t)F(0)\rangle. \tag{53}$$

The equality of the two correlators in Eq. (53) follows from the fact that the symmetry transformation $\hat{n} \to -\hat{n}$, $\phi_{R\alpha} \to -\phi_{R\alpha}$, and $N \to -N$ does not affect the Hamiltonian (46), (47), and (49), but changes $F^\dagger \leftrightarrow F$.

It is instructive to apply the formula (52) to the noninteracting case, $E_C = 0$. In this limit the operators F and $F^\dagger$ commute with the Hamiltonian (46) and (49). Thus the correlator (53) is unity, and we get $G = G_L$. This is the expected result, since we assumed $G_L \to 0$, and the conductance of the system must be determined by that of the weakest link.

Let us now find the conductance at nonzero charging energy E_C, but in the absence of the scattering in the right contact, $r_R = 0$. In this case the correlator (53) is no longer trivial, because the operators F and $F^\dagger$ do not commute with the interaction term (47). When an electron tunnels into the dot, $n \to n+1$, the system is shifted from its equilibrium state. To accommodate the additional electron brought by the tunneling process and to return the system to the equilibrium state, one electron must leave the dot through the right contact. Thus the time evolution of the operators F and $F^\dagger$ becomes nontrivial.

To find the correlator $K(t)$, we will perform the following unitary transformation of the Hamiltonian:

$$U = e^{i\hat{n}\Theta}, \tag{54}$$

$$\Theta = \frac{\pi}{2}\int_{-\infty}^{\infty}[\Pi_{R\uparrow}(y) + \Pi_{R\downarrow}(y)]\,dy. \tag{55}$$

This transformation shifts the bosonic fields $\phi_{R\alpha}$ and adds a phase factor to the charge-lowering operator F,

$$\phi_{R\alpha}(x) \to \phi_{R\alpha}(x) - \frac{\pi}{2}\hat{n}, \tag{56}$$

$$F \to Fe^{i\Theta}. \tag{57}$$

One can easily see that the kinetic-energy term (46) is invariant under the transformation (54), whereas the charging term (47) transforms to an $\hat{n}$-independent form,

$$\tilde{H}_C = U^\dagger H_C U = E_C\left[\frac{1}{\pi}\sum_\alpha \phi_{R\alpha}(0) - N\right]^2. \tag{58}$$

Upon this transformation, the operators F and $F^\dagger$ commute with the Hamiltonian (46) and (58), but they acquire nontrivial phase factors; see Eq. (57). This enables us to find $K(t)$ as the correlator of the phase factors, $K(t) = K_\Theta(t) \equiv \langle e^{i\Theta(t)}e^{-i\Theta(0)}\rangle$. Since the phase Θ is linear in bosonic variables, the calculation of the correlator K_Θ is straightforward:

$$\begin{aligned} K_\Theta(t) &= \exp\{-\langle[\Theta(0) - \Theta(t)]\Theta(0)\rangle\} \\ &\simeq \frac{\pi^2 T}{2i\gamma E_C}\frac{1}{\sinh[\pi T(t - i\delta)/\hbar]}. \end{aligned} \tag{59}$$

The last equality is valid asymptotically at low energies, $T, \hbar/t \ll E_C$. The substitution of $K = K_\Theta$ into the expression for the conductance (52) leads to a linear temperature dependence of the conductance:

$$G = G_L\frac{\pi^3 T}{8\gamma E_C}. \tag{60}$$

Physically the origin of the linear suppression of the conductance at low temperatures (60) is the orthogonality catastrophe.[29] Unlike the noninteracting case $E_C = 0$ discussed above, the charge fluctuations in the dot are now suppressed at energies below E_C. Effectively one can interpret the effect of the charging energy as a hard-wall boundary condition for the wave functions of the right-contact electrons with energies within the band of width $W \sim E_C$ near the Fermi level. When an electron tunnels through the left barrier, the system must lower its charging energy by moving one electron through the right contact. Since the two spin channels are completely symmetric, each mode transfers the charge $q = \frac{e}{2}$. According to the Friedel sum rule, this leads to a change in the boundary condition corresponding to an additional scattering phase shift $\delta = \pm\frac{\pi q}{e} = \pm\frac{\pi}{2}$. A sudden change of the boundary conditions creates a state with a large number of electron-hole excitations, which is nearly orthogonal[29] to the ground state of the system. This orthogonality leads to a power-law suppression of the tunneling density of states $\nu(\varepsilon) \propto \varepsilon^\chi$, with the exponent[30–32] determined by the phase shifts in all the electronic channels, $\chi = \sum(\delta/\pi)^2$. In our system, each spin mode gives two channels—in the dot and in the right lead—leading to the total of four channels. Thus the exponent χ is unity, and the density of states $\nu(\varepsilon) \propto \varepsilon$. The linear suppression of the tunneling density of states results in the T-linear conductance (60).

The linear temperature dependence of conductance

(60) appears to contradict the T^2 law for the inelastic cotunneling, Eqs. (3), (11), and (39). The reason is the absence of the barrier in the right junction assumed in the derivation of Eq. (60). We will now show that the presence of arbitrarily weak barrier in the right contact is crucial (see also Appendix A 3) and leads to the quadratic temperature dependence of the conductance at $T \to 0$.

The weak backscattering in the right contact is described by the term (49) in the Hamiltonian. As follows from Eq. (56), under the unitary transformation (54) the scattering term (49) acquires an additional sign:

$$\tilde{H}'_R = (-1)^{\hat{n}} \frac{1}{\pi} D|r_R| \sum_\alpha \cos[2\phi_{R\alpha}(0)]. \tag{61}$$

The operators F and $F^\dagger$ no longer commute with the Hamiltonian: they commute with $H_0 + \tilde{H}_C$ and anticommute with $\tilde{H}'_R$. Therefore the correlator (53) is no longer equal to K_Θ, but rather $K(t) = K_\Theta(t) K_F(t)$, where

$$K_F(t) = \langle F(t) F^\dagger(0) \rangle. \tag{62}$$

Here the averaging $\langle\rangle$ is over the equilibrium thermal distribution of the transformed Hamiltonian, $H = H_0 + \tilde{H}_C + \tilde{H}'_R$.

In the absence of the backscattering in the right contact, K_Θ is given by Eq. (59), and $K_F(t) \equiv 1$. When a weak backscattering is added, both correlators are affected. It is clear, however, that K_Θ is not modified significantly. Indeed, it follows from the above-mentioned analogy with the orthogonality catastrophe that the time dependence (59) is determined solely by the charge transferred through the right junction, which in our case is always e—the charge of the electron brought into the dot through the left barrier—and is not affected by the presence of the right barrier. Therefore the only change in $K_\Theta(t)$ caused by the weak barrier can be a small modification of the prefactor in Eq. (59). On the other hand, we will show that $K_F(t)$ is modified significantly, which leads to the quadratic temperature dependence of the conductance.

To find $K_F(t)$ we first note that the Hamiltonian $H = H_0 + \tilde{H}_C + \tilde{H}'_R$ is identical to the one considered in Sec. III and in Ref. 15. The difference with the problem considered in Sec. III is that instead of two modes corresponding to two different contacts we now have two spin modes in the same (right) contact. Thus the two scattering amplitudes are equal and given by $|r_R|$. As we have seen in Sec. III, the presence of the backscattering shows up at low-energy scales $\epsilon \sim |r_R|^2 E_C \ll E_C$, where one can effectively integrate out the mode related to the charge fluctuations in the dot, $\phi_{R\uparrow} + \phi_{R\downarrow}$. The resulting Hamiltonian is expressed in terms of the spin field $\phi_{Rs} = (\phi_{R\uparrow} - \phi_{R\downarrow})/\sqrt{2}$ and has the form

$$H = \frac{\hbar v_F}{2} \int \left\{ \frac{1}{\pi} [\nabla \phi_{Rs}(x)]^2 + \pi \Pi_{Rs}^2(x) \right\} dx + (-1)^{\hat{n}} \sqrt{\frac{8\gamma E_C D}{\pi^3}} |r_R| \cos \pi N \cos\left[\sqrt{2}\phi_{Rs}(0)\right]. \tag{63}$$

We can now fermionize the Hamiltonian using a representation of the operators $e^{\pm i\sqrt{2}\phi_{Rs}}$ in terms of the fermion creation and annihilation operators in a way identical to the one leading from the Hamiltonian (29) and (30) to Eq. (35). The final Hamiltonian is again quadratic in fermion operators, but also depends on the operator $\hat{n}$ of the number of particles transferred through the left barrier,

$$H = i\hbar v_F \int \psi^\dagger(x) \nabla \psi(x) dx + (-1)^{\hat{n}} \frac{2}{\pi} \sqrt{\gamma \hbar v_F E_C}\, |r_R| \cos \pi N \times (c + c^\dagger)[\psi(0) - \psi^\dagger(0)]. \tag{64}$$

Here we presented a Majorana fermion operator similar to η_+ in Eq. (35) as a sum of fermion creation and annihilation operators, $\eta = c + c^\dagger$. This representation is more convenient, because we can now replace the operators F and $F^\dagger$ in the definition (62) of the correlator K_F by $F_s = F_s^\dagger = 1 - 2c^\dagger c$. Indeed, these new operators commute with the first term in the Hamiltonian (64), anticommute with the second one, and possess the property $F_s F_s^\dagger = F F^\dagger = 1$. Therefore one can find the correlator K_F as[33]

$$K_F(t) = \langle [1 - 2c^\dagger(t) c(t)][1 - 2c^\dagger(0) c(0)] \rangle. \tag{65}$$

The actual calculation of the correlator (65) is now straightforward, as the Hamiltonian (64) is quadratic in the fermion operators. The result for the correlator K_F has the form

$$K_F(t) = \frac{2\Gamma_R}{\pi} \int_{-\infty}^{\infty} \frac{e^{iEt} dE}{(E^2 + \Gamma_R^2)(e^{E/T} + 1)}. \tag{66}$$

Here we have introduced the low-energy scale

$$\Gamma_R = \frac{8\gamma}{\pi^2} E_C |r_R|^2 \cos^2 \pi N, \tag{67}$$

originating from the presence of a weak scatterer in the right contact. In the weak-scattering limit, $\Gamma_R \to 0$, Eq. (66) reproduces the result $K_F(t) = 1$ mentioned above. However, in the opposite limit $T, \hbar/t \ll \Gamma_R$, the correlator K_F has the same nontrivial time dependence as the correlator $K_\Theta(t)$ given by Eq. (59),

$$K_F(t) \simeq \frac{2T}{i\Gamma_R} \frac{1}{\sinh[\pi T(t - i\delta)/\hbar]}.$$

This additional time-dependent contribution to $K(t)$ leads to the quadratic temperature dependence of the linear conductance at $T \ll \Gamma_R$.

To find the conductance at temperatures $T \sim \Gamma_R$, we can now substitute the correlator $K(t) = K_\Theta(t) K_F(t)$ into Eq. (52), which leads to the following expression:

$$G = \frac{T G_L}{8\gamma E_C} \int_{-\infty}^{\infty} \frac{\Gamma_R}{E^2 + \Gamma_R^2} \frac{\pi^2 + (E/T)^2}{\cosh^2(E/2T)} dE. \tag{68}$$

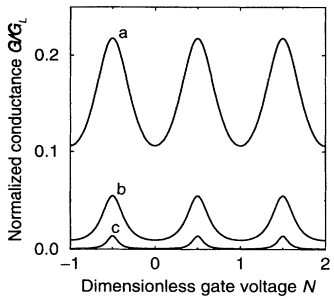

FIG. 3. Conductance (68) as a function of the dimensionless gate voltage N for the asymmetric case, $|r_R| = 0.4$. The three curves are calculated for $E_C/T =$ (a) 10, (b) 40, and (c) 160.

Equation (68) is the central result of this section. It is valid for any T/Γ_R, if both temperature and Γ_R are much smaller than the charging energy E_C. The condition $\Gamma_R \ll E_C$ means $|r_R| \ll 1$, i.e., the scattering in the right contact must indeed be weak.

In the limit $T \gg \Gamma_R$, Eq. (68) reproduces the previously obtained linear temperature dependence (60) in the absence of the scattering in the right contact. It is important to note, however, that the linear temperature dependence is obtained not only at $r_R = 0$, but also in the presence of the barrier if N is half-integer, i.e., at the centers of conductance peaks.

Away from the centers of the peaks, the conductance has a stronger temperature dependence, $G \propto T^2$. Indeed, at $T \ll \Gamma_R$ the expression (68) reduces to

$$G = \frac{2\pi^2}{3\gamma}\frac{T^2}{E_C\Gamma_R}G_L. \tag{69}$$

Thus the quadratic temperature dependence of the linear conductance is restored for the off-peak values of the gate voltage. The evolution of the conductance peaks is illustrated in Fig. 3.

V. SCALING APPROACH TO INELASTIC COTUNNELING

In Secs. II–IV we have considered several cases when the inelastic cotunneling can be studied analytically even though one or both contacts are in the strong-tunneling regime. A common property of all these results is that at $T \to 0$ the conductance behaves as T^2 away from the resonance values of the gate voltage, i.e., at $N \neq \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$. The same temperature dependence was obtained earlier[5] for the off-resonance conductance in the weak-tunneling case; see Eq. (3). In this section we study the general scaling properties of our system and argue that the quadratic temperature dependence of linear conductance is universal and should hold for arbitrary barriers.

We will describe our system by the bosonized Hamiltonian (19)–(21) in the spinless case and (40)–(42) in the presence of spins. In the absence of the backscattering the Hamiltonian is quadratic, i.e., $|r_L| = |r_R| = 0$ is a fixed point of our Hamiltonian. In analogy with the Kondo problem we will call it the strong-coupling fixed point. To investigate the stability of this fixed point, one should find the scaling dimension of the backscattering term (21) or (42).

We shall start with the spinless case, where at $T = 0$ and $t \gg \hbar/E_C$ a simple calculation gives

$$\langle H'(t)H'(0)\rangle = \frac{\hbar\Gamma_0(N)}{2\pi i t} + \frac{\gamma\hbar^3(|r_L|^2 + |r_R|^2 - 2|r_L||r_R|\cos 2\pi N)}{4\pi i E_C t^3},$$

with $\Gamma_0(N)$ defined by Eq. (26). The long-time asymptotics of the correlator $\langle H'(t)H'(0)\rangle$ is given by the first term, which is proportional to t^{-1}, unless $\Gamma_0 = 0$. Thus the dimension of the perturbation H' is $\frac{1}{2} < 1$, and the strong-coupling fixed point is unstable. At a half-integer N and in a perfectly symmetric case, $|r_L| = |r_R|$, the parameter Γ_0 vanishes, and the dimension of the operator H' becomes $\frac{3}{2} > 1$. In this case the strong-coupling fixed point is stable. In the language of the corresponding two-channel Kondo problem this means that the strong-coupling fixed point is stable only in the absence of magnetic field and channel anisotropy. The leading irrelevant operator with dimension $\frac{3}{2}$ gives rise to the weak temperature dependence of the peak heights in the symmetric spinless case; see Appendix A 1.

A similar analysis can be performed in the case of electrons with spins, described by the Hamiltonian (40)–(42). The correlator of the perturbation operators now has the form

$$\langle H'(t)H'(0)\rangle = \left(|r_L|^2 + |r_R|^2\right)\sqrt{\frac{4\gamma E_C}{\pi^5}}\left(\frac{\hbar}{it}\right)^{3/2},$$

meaning that H' has dimension $\frac{3}{4} < 1$ at any N and any barrier strengths. Thus as expected the strong-coupling fixed point in the four-channel case is unstable. At a half-integer N this statement has a well-known analog in the theory[27] of the multichannel Kondo problem, where the strong-coupling fixed point is unstable even in the absence of the magnetic field.

The fact that the strong-coupling fixed point is unstable means that as the temperature is lowered, the weak backscattering in the contacts becomes stronger. It is natural to assume that the effective barriers grow indefinitely, and the system approaches the weak-coupling fixed point. To test this hypothesis we will consider a system where the two contacts are in the weak-tunneling regime and study the stability of the corresponding fixed point.

As we saw in Secs. II and IV, in the weak-tunneling case it is convenient to describe the system in terms of the original fermionic variables. Generalizing the Hamiltonian of Sec. IV B to the case of both barriers being in the weak-tunneling regime, we introduce the Hamiltonian $H = H_0 + H_C + H'_L + H'_R$, where

$$H_0 = \sum_{k\alpha} \epsilon_k a^\dagger_{k\alpha} a_{k\alpha} + \sum_{p\alpha} \epsilon_p a^\dagger_{p\alpha} a_{p\alpha} + \sum_{q\alpha} \epsilon_q a^\dagger_{q\alpha} a_{q\alpha} + \sum_{s\alpha} \epsilon_s a^\dagger_{s\alpha} a_{s\alpha}, \tag{70}$$

$$H_C = E_C(\hat{n}_L + \hat{n}_R - N)^2, \tag{71}$$

$$H'_L = \sum_{kp\alpha} (v_L a^\dagger_{k\alpha} a_{p\alpha} F_L + v^*_L a^\dagger_{p\alpha} a_{k\alpha} F^\dagger_L), \tag{72}$$

$$H'_R = \sum_{qs\alpha} (v_R a^\dagger_{s\alpha} a_{q\alpha} F_R + v^*_R a^\dagger_{q\alpha} a_{s\alpha} F^\dagger_R). \tag{73}$$

Here $a_{k\alpha}$ and $a_{s\alpha}$ describe the electrons in the left and right leads, respectively; $a_{p\alpha}$ and $a_{q\alpha}$ correspond to the electrons in the dot coupled to the left and right contacts. Similarly to the formalism of Sec. IV B, we have introduced the operators $\hat{n}_L$ and $\hat{n}_R$ of the number of electrons tunneled through the left and right contacts and the corresponding raising and lowering operators F_L, $F^\dagger_L$, F_R, and $F^\dagger_R$ in the tunnel Hamiltonians (72) and (73).

Away from the resonance values of the gate voltage, $N \neq \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$, the states with different n_L and n_R have different energies because of the Coulomb term (71). Since the tunnel Hamiltonian given by (72) and (73) changes n_L or n_R, it does not describe the transitions between the low-energy states of the Hamiltonian. Thus when considering the low-energy properties of the system, one should use the tunneling terms (72) and (73) to construct the perturbation that is not accompanied by the change of the total charge of the dot. In the simplest case, this can be done by selecting the terms containing an equal number of operators F_i and $F^\dagger_i$ in the second-order perturbation

$$H'_2 = (H'_L + H'_R) \frac{1}{E_0 - H_0 - H_C} (H'_L + H'_R). \tag{74}$$

Here the denominator accounts for the energy of the virtual state created by adding (or removing) an electron to the dot. At very low energies the only important contribution to the denominator is the change of the electrostatic energy of the system.

At N in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ a typical representative of the variety of terms contained in Eq. (74) has the form

$$A = -v^*_L v_R \left[\frac{1}{E_C(1-2N)} + \frac{1}{E_C(1+2N)}\right] \times F^\dagger_L F_R \sum_{kp\alpha} a^\dagger_{p\alpha} a_{k\alpha} \sum_{qs\alpha'} a^\dagger_{s\alpha'} a_{q\alpha'}. \tag{75}$$

The terms included in Eq. (75) correspond to the processes where one electron tunnels into the dot through the left junction and another one escapes from the dot through the right junction. Depending on which of the two tunneling events is executed first, we get different energies of the virtual states, corresponding to the two denominators in Eq. (75).

To find out whether or not the weak-coupling fixed point is stable, one should evaluate the scaling dimension of operator A and other similar contributions in Eq. (74). A straightforward calculation gives

$$\langle A(t) A^\dagger(0) \rangle = \frac{\hbar^2 G_L G_R}{(2\pi e^2)^2} \left[\frac{1}{E_C(1-2N)} + \frac{1}{E_C(1+2N)}\right]^2 \times \left\{\frac{\pi T}{\sinh[\pi T(t - i\delta)/\hbar]}\right\}^4. \tag{76}$$

At $T \to 0$ the correlator decays as $1/t^4$, and therefore the dimension of the operator A is 2. One can also check that the other contributions to Eq. (74) have the same dimension, whereas the higher-order terms have even higher dimensions. Thus the weak-coupling fixed point is stable.

So far we have shown that away from the resonance values of the gate voltage, $N \neq \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$, the strong-coupling fixed point is unstable whereas the weak-coupling fixed point is stable. We now conjecture that there are no fixed points at intermediate coupling. Then at low energies the system always evolves towards the stable (weak-coupling) fixed point. One should also expect that near this fixed point the temperature dependence of conductance is determined by the dimension of the leading irrelevant perturbation, which is a universal characteristic of the problem. Indeed, one can easily show that the leading contribution to the current operator $I = e\dot{n}_L$ is given by

$$I = -\frac{ie}{\hbar}(A - A^\dagger). \tag{77}$$

We now substitute this current operator into the Kubo formula (22), and use (76) to find the following expression for the conductance:

$$G = \frac{\pi\hbar G_L G_R}{3e^2} \left[\frac{1}{E_C(1-2N)} + \frac{1}{E_C(1+2N)}\right]^2 T^2. \tag{78}$$

This expression for the conductance in the weak-tunneling limit coincides with the well-known result for the inelastic cotunneling,[5] and reproduces Eq. (3) near the resonances, $\frac{1}{2} \pm N \ll 1$.

It is important to note that in the derivation of Eq. (78) we neglected all the contributions to the current operator with dimensions $d > 2$. Such terms would lead to the additional terms in the correlator (76), which are proportional to $\{\pi T/\sinh[\pi T(t - i\delta)/\hbar]\}^m$ with $m > 4$. Clearly, the corresponding corrections to the conductance would behave as $\delta G \propto T^{m-2} \ll T^2$. Thus at $T \to 0$ the temperature dependence of the conductance is given by the leading irrelevant perturbation. As we have seen, the universal scaling dimension $d = 2$ of this perturbation translates into the universal temperature dependence of the conductance $G \propto T^2$. Therefore this temperature dependence is specific not only for the cases of weak or strong tunneling, but should hold for any barrier strengths.

The above treatment of the stability of the weak-coupling fixed point was limited to the off-resonance values of the gate voltage. We will now demonstrate that the properties of the system are completely different at half-integer N. Consider, for example, the case of $N = \frac{1}{2}$. Then the states with the dot charge equal to 0 and e have the same energy. As a result the terms in the tunnel Hamiltonians (72) and (73) responsible for the transitions between these two states no longer lead to the large increase in the energy of the system and should

not be discounted. It is easy to check that the dimension of such operators is $d = 1$; i.e., the tunnel Hamiltonian is a marginal operator. This conclusion is actually obvious, because the same calculation of scaling dimension must be valid in the absence of the charging effects, $E_C = 0$, where we do not expect the tunneling amplitudes to be renormalized as the temperature is lowered. However, the simple calculation of the scaling dimension is not sufficient to determine the stability of the weak-coupling fixed point. To make such a determination, one has to study the scaling equations for the tunneling amplitudes including the higher-order terms in v_L and v_R. We have performed this analysis in Sec. II, where we saw from the mapping to the multichannel Kondo problem that the tunneling amplitude is in fact a marginally relevant perturbation; i.e., it grows logarithmically at low temperatures. Therefore on resonance the weak-coupling fixed point is unstable.

The on-resonance scaling properties of our system are strongly affected by the presence of electron spins and by the symmetry of the two barriers. Let us first consider the case of symmetric barriers. In the spinless case we saw that on resonance the strong-coupling fixed point is stable if the barriers are symmetric. Thus one should expect the system to always renormalize towards this fixed point. Then the conductance must be on the order of $e^2/\hbar$. This is confirmed by the explicit calculation in the strong-tunneling case, Eq. (38).

In the presence of spins, both weak- and strong-coupling fixed points are unstable on resonance. Thus one should expect that there must be a stable fixed point at some intermediate coupling strength. This hypothesis is also confirmed by the analogy with the four-channel Kondo problem discussed in Sec. II. Indeed, it is well known that the low-energy properties of the Kondo problem with the number of channels exceeding 2 is governed by an intermediate-coupling fixed point, which gives rise to a specific non-Fermi-liquid behavior of the susceptibility and specific heat.[27,34–36] In Appendix B we show how one can study the properties of the intermediate-coupling fixed point starting from the vicinity of the strong-coupling point. We discover that on resonance the thermodynamics of our problem is indeed identical to that of the four-channel Kondo problem.

The technique presented in Appendix B, however, does not enable us to find the low-temperature conductance G. Nevertheless one can estimate the order of magnitude of G at the intermediate-coupling fixed point in the following way. From the fact that the strong-coupling fixed point is unstable and from the result (45) of the perturbation theory near this point we know that the conductance must be smaller than $e^2/2\pi\hbar$. On the other hand, at $G \ll e^2/\hbar$ the weak-coupling treatment of Sec. II is applicable and predicts the growth of conductance at $T \to 0$. Therefore the limiting value of conductance must be of the order of, but smaller than, $e^2/2\pi\hbar$. We further conjecture that the peak conductance is a universal characteristic of the intermediate-coupling fixed point (see Appendix B), meaning that in the limit $T \to 0$ the peak conductance does not depend on the heights of the barriers.

The scaling analysis in the spirit of Ref. 23 enables us to make some conclusions about the shape of the peaks in $G(N)$. In the spinless case we see from Eq. (38) that at the tails of the peak at $N = \frac{1}{2}$ the conductance decays as $G \propto 1/\left(N - \frac{1}{2}\right)^4$. This result can be interpreted within the scaling picture as follows. A small deviation of N from $\frac{1}{2}$ is identified as a magnetic field h in the appropriate multichannel Kondo problem; see Eq. (7). Therefore such a deviation is a relevant perturbation with scaling dimension $d = 2/(2+k)$, where k is the number of channels.[35] In the spinless case we have $k = 2$ and $d = \frac{1}{2}$. This means that as the temperature is lowered, the magnetic field grows as $h(T) \propto \left(N - \frac{1}{2}\right)T^{-1/2}$. We now conjecture that near the fixed point, at $T \to 0$, the conductance is a universal function of h only, $G = G(h(T))$. As we discussed above, away from the center of the peak $G \propto T^2$ at $T \to 0$. Therefore the magnetic field dependence of the conductance must be given by $G \propto 1/h^4 \propto T^2/\left(N - \frac{1}{2}\right)^4$, in agreement with Eq. (38). We can now apply the same argument to the case of electrons with spins, where the number of channels $k = 4$ and the scaling dimension of the magnetic field is $d = \frac{1}{3}$. The temperature dependence of the renormalized magnetic field is consequently $h(T) \propto \left(N - \frac{1}{2}\right)T^{-2/3}$. Therefore

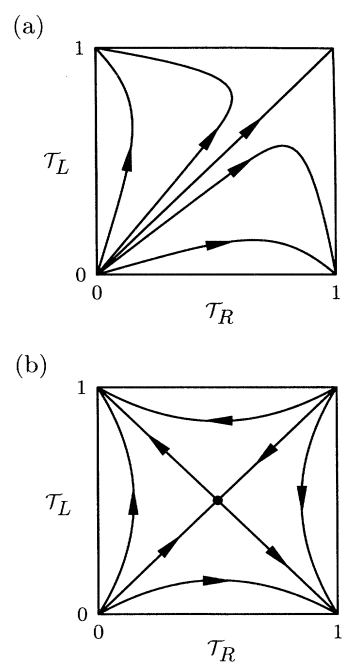

FIG. 4. Scaling diagram of the two-barrier system in the cases of (a) spinless electrons and (b) electrons with spins. The gate voltage corresponds to a resonance value, $N = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$. In the symmetric case, $\mathcal{T}_L = \mathcal{T}_R$, the system scales to either strong- or intermediate-coupling fixed point, depending on the number of channels. In an asymmetric case the system approaches the limit where the weaker barrier disappears, while the stronger one becomes very high.

near the intermediate-coupling fixed point the condition $G \propto T^2$ at $T \to 0$ demands $G \propto h^{-3} \propto T^2/(N-\frac{1}{2})^3$. Thus we expect the tails of the conductance peaks to decay as $1/(N-\frac{1}{2})^3$.

So far we discussed the case of symmetric barriers. The on-resonance behavior of the system is very different if the two barriers are not identical. This can be readily seen from the analogy to the Kondo problem. From the theory of the multichannel Kondo model[27,37] it is known that the asymmetry between the channels is a relevant perturbation. In terms of the Coulomb blockade problem this means that on resonance a small asymmetry will grow and eventually the smaller transmission coefficient, say $\mathcal{T}_L$, will scale to zero, whereas the larger one, $\mathcal{T}_R$, must approach the fixed point of the problem with half the number of channels of the original one. Thus for both two- and four-channel cases $\mathcal{T}_R \to 1$. For the most realistic case of electrons with spins, near this asymmetric fixed point the system is identical to the one considered in Sec. IV B. Thus we conclude that for asymmetric systems (i) the on-resonance conductance vanishes linearly in T, and (ii) the shape of the resonances in $G(N)$ is universal and given by Eq. (68).

The on-resonance scaling properties of our problem are summarized in Fig. 4.

VI. CONCLUSION

In this paper we have studied the transport through a quantum dot connected to two leads by quantum point contacts, Fig. 1. Unlike in conventional theories of Coulomb blockade,[1,2] we concentrate on the regime of strong tunneling, where one or both contacts are close to perfect transmission. We find the linear conductance in a wide interval of temperatures, limited by the level spacing in the dot from below and by the charging energy E_C from above.

We have shown that as long as there is any backscattering in the contacts, i.e., both transmission coefficients $\mathcal{T}_{L,R} < 1$, the Coulomb blockade peaks in the conductance as a function of the gate voltage should be observed. Between the Coulomb blockade peaks the temperature dependence of the conductance at $T \to 0$ is determined by the weak-coupling fixed point. As a result we get the T^2 dependence of the conductance, which coincides with the known result[5] for the inelastic cotunneling mechanism.

Our results show that the behavior of the conductance near the peaks depends qualitatively on the presence of electron spins. In the spinless case the strong-tunneling fixed point is stable if the barriers are symmetric, and thus the technique developed in this paper allows a complete solution of the problem. The resulting peaks in conductance have the height $e^2/4\pi\hbar$, with the shape given by Eq. (38). In the presence of the spins, the strong-tunneling fixed point is no longer stable, and our technique does not allow a complete solution of the problem. Using a useful analogy with the multichannel Kondo problem, we have concluded that the peak conductance must still be of the order of $e^2/\hbar$. In addition, we studied the peaks in conductance in the case of asymmetric barriers. The peak conductance in this case is linear in temperature at $T \to 0$, and its shape is given by Eq. (68).

The strong-tunneling regime of Coulomb blockade was explored to some extent in several recent experiments.[6–9] Our results are in qualitative agreement with the experiment.[6] We believe that our predictions can be easily tested in similar experiments.

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APPENDIX A: PERTURBATION THEORY FOR WEAK REFLECTION

In this appendix we derive the leading-order correction to the conductance due to the weak reflection at the quantum point contacts. We shall use a path integral method to calculate the current-current correlation function at imaginary frequencies and then perform analytic continuation to the real frequencies.

1. Spinless fermions

In the spinless case our problem is similar to the resonant tunneling problem studied in Ref. 21. Thus we can apply the method developed in Refs. 21 and 38 to calculate the conductance perturbatively with respect to the reflection coefficients.

The current operator is given by $I = (e/\sqrt{2}\pi)\partial_t\phi_I(0,t)$. Then the Kubo formula (22) for the conductance can be rewritten as

$$G = \frac{e^2 T}{2\pi^2 i\hbar^2} \lim_{\omega\to 0} \omega \lim_{i\omega_n \to \omega + i0^+} \langle \phi_I(-i\omega_n)\phi_I(i\omega_n)\rangle, \quad \text{(A1)}$$

where $\omega_n = 2\pi n T/\hbar$ and

$$\phi_I(i\omega_n) = \int_0^{\hbar/T} e^{i\omega_n \tau}\phi_I(0,\tau)d\tau. \quad \text{(A2)}$$

The thermal average $\Phi_I(i\omega_n) \equiv \langle \phi_I(-i\omega_n)\phi_I(i\omega_n)\rangle$ is calculated as

$$\Phi_I(i\omega_n) = \frac{1}{Z_J^{(1)}} \frac{\delta^2 Z_J^{(1)}}{\delta J(i\omega_n)\delta J(-i\omega_n)}\bigg|_{J=0}, \quad \text{(A3)}$$

where the generating functional is given by

$$Z_J^{(1)} = \int \mathcal{D}\phi_I(x,\tau) \int \mathcal{D}\phi_C(x,\tau) \exp\left[-\frac{S_1}{\hbar} + \sum_{\omega_n} J(-i\omega_n)\phi_I(i\omega_n)\right],$$

$$S_1 = \frac{\hbar}{2\pi}\int_0^{\hbar/T} d\tau \int dx \left[\frac{1}{v_F}(\partial_\tau\phi_I)^2 + v_F(\partial_x\phi_I)^2 + \frac{1}{v_F}(\partial_\tau\phi_C)^2 + v_F(\partial_x\phi_C)^2\right] + \frac{2E_C}{\pi^2}\int_0^{\hbar/T} d\tau \left[\phi_C(0,\tau) - \frac{\pi N}{\sqrt{2}}\right]^2$$
$$+\frac{D}{\pi}\int_0^{\hbar/T} d\tau \Big(|r_L|\cos\{\sqrt{2}\,[\phi_I(0,\tau) - \phi_C(0,\tau)]\} + |r_R|\cos\{\sqrt{2}\,[\phi_I(0,\tau) + \phi_C(0,\tau)]\}\Big). \tag{A4}$$

Expanding $\exp(-S_1/\hbar)$ in powers of $|r|$ and performing Gaussian integrals, we get

$$\Phi_I(i\omega_n) = \frac{\pi\hbar}{2|\omega_n|T} - \frac{\gamma E_C}{2\hbar\omega_n^2}\left[|r_L|^2 + |r_R|^2 + 2|r_L||r_R|\cos(2\pi N)\right]\int_0^{\hbar/T}\frac{1-\cos\omega_n\tau}{\sin(\pi T\tau/\hbar)}d\tau$$
$$-\frac{\pi^4\gamma T^2}{8\hbar\omega_n^2 E_C}\left[|r_L|^2 + |r_R|^2 - 2|r_L||r_R|\cos(2\pi N)\right]\int_0^{\hbar/T}\frac{1-\cos\omega_n\tau}{\sin^3(\pi T\tau/\hbar)}d\tau + O(|r|^4). \tag{A5}$$

In this calculation we have used the following relations:

$$\frac{T}{\hbar}\sum_{\omega_n'}\frac{e^{-\hbar|\omega_n'|/D}}{|\omega_n'|}(1-\cos\omega_n'\tau) = \frac{1}{\pi}\ln\left[\frac{D}{\pi T}\sin\left(\frac{\pi T\tau}{\hbar}\right)\right], \tag{A6a}$$

$$\frac{T}{\hbar}\sum_{\omega_n'}\frac{e^{-\hbar|\omega_n'|/D}}{|\omega_n'| + (2E_C/\pi\hbar)} = \frac{1}{\pi}\ln\left(\frac{\pi D}{2\gamma E_C}\right), \tag{A6b}$$

$$\frac{T}{\hbar}\sum_{\omega_n'}\frac{e^{-\hbar|\omega_n'|/D}}{|\omega_n'| + (2E_C/\pi\hbar)}\cos\omega_n'\tau = \frac{\pi^3T^2}{4E_C^2}\frac{1}{\sin^2(\pi T\tau/\hbar)}, \tag{A6c}$$

where we have neglected higher-order terms in T/E_C. Note that Eqs. (A6a) and (A6c) are correct only for $\tau, \hbar T^{-1} - \tau \gg \hbar/D$.

In calculating the integrals over τ in Eq. (A5), we introduce a short-time cutoff $\tau_c \sim \hbar/D$, which is omitted in Eqs. (A6a) and (A6c), and modify the contour of integration:

$$\int_0^{\hbar/T}\frac{1-\cos\omega_n\tau}{\sin^\nu(\pi T\tau/\hbar)}d\tau = \int_{\tau_c}^{\hbar/T-\tau_c}\frac{1-e^{i\omega_n\tau}}{\sin^\nu(\pi T\tau/\hbar)}d\tau$$
$$= i\int_0^\infty\frac{1-e^{i\omega_n(\tau_c+it)}}{\sin^\nu[\pi T(\tau_c+it)/\hbar]}dt - i\int_0^\infty\frac{1-e^{i\omega_n(-\tau_c+it)}}{\sin^\nu[\pi T(\tau_c-it)/\hbar]}dt. \tag{A7}$$

By replacing $i\omega_n$ by ω and taking the limit $\omega \to 0$, Eq. (A7) is reduced to $-i\omega F(T,\nu)$, where

$$F(T,\nu) \equiv \int_{-\infty}^\infty\frac{\tau_c+it}{\sin^\nu[\pi T(\tau_c+it)/\hbar]}dt = \frac{\hbar^2}{2\pi T^2}\int_{-\infty}^\infty\frac{dx}{\cosh^\nu x} = \frac{\hbar^2}{2\pi T^2}\frac{\sqrt{\pi}\,\Gamma(\frac{\nu}{2})}{\Gamma(\frac{\nu}{2}+\frac{1}{2})}. \tag{A8}$$

From Eqs. (A1), (A5), and (A8) we finally obtain

$$G = \frac{e^2}{4\pi\hbar}\left\{1 - \frac{\gamma E_C}{2\pi T}\left[|r_L|^2 + |r_R|^2 + 2|r_L||r_R|\cos(2\pi N)\right] - \frac{\pi^3\gamma T}{16E_C}\left[|r_L|^2 + |r_R|^2 - 2|r_L||r_R|\cos(2\pi N)\right]\right\}. \tag{A9}$$

The term proportional to T/E_C originates from the weak fluctuations of the charge of the dot at low frequencies. At a half-integer N it is related to the leading irrelevant operator near the fixed point of the two-channel Kondo problem. The scaling dimension of this operator is $d = \frac{3}{2}$ (see Sec. V), yielding the temperature dependence $\delta G \propto T^{2d-2} = T$.

2. Electrons with spins

The calculation proceeds in parallel to the spinless case. We first define the new sets of phase fields,

$$\phi_{Lc} = \frac{1}{\sqrt{2}}(\phi_{L\uparrow} + \phi_{L\downarrow}), \quad \phi_{Ls} = \frac{1}{\sqrt{2}}(\phi_{L\uparrow} - \phi_{L\downarrow}), \tag{A10}$$

$$\phi_{Rc} = \frac{1}{\sqrt{2}}(\phi_{R\uparrow} + \phi_{R\downarrow}), \quad \phi_{Rs} = \frac{1}{\sqrt{2}}(\phi_{R\uparrow} - \phi_{R\downarrow}),$$

where ϕ_{Lc} and ϕ_{Rc} describe the charge-density fluctuations, and ϕ_{Ls} and ϕ_{Rs} correspond to the spin-density fluctuations. We further define their momentum conjugates Π_{Lc}, Π_{Ls}, Π_{Rc}, and Π_{Rs} in the same way.

The current operator in this basis is $I = (e/\sqrt{2}\pi)\partial_t[\phi_{Lc}(0,t) + \phi_{Rc}(0,t)]$, and accordingly the Kubo formula for the conductance is

$$G = \frac{e^2 T}{2\pi^2 i\hbar^2} \lim_{\omega\to 0} \omega \lim_{i\omega_n \to \omega + i0^+} \Phi_c(i\omega_n), \tag{A11}$$

where $\Phi_c(i\omega_n)$ is given by

$$\Phi_c(i\omega_n) = \frac{1}{Z_J^{(2)}} \frac{\delta^2 Z_J^{(2)}}{\delta J(-i\omega_n)\delta J(i\omega_n)}\bigg|_{J=0} \tag{A12}$$

with the generating functional,

$$Z_J^{(2)} = \int \mathcal{D}\phi_{Lc}(x,\tau) \int \mathcal{D}\phi_{Ls}(x,\tau) \int \mathcal{D}\phi_{Rc}(x,\tau) \int \mathcal{D}\phi_{Rs}(x,\tau) \exp\left[-\frac{1}{\hbar}(S_2 + S_J)\right], \tag{A13}$$

$$\begin{aligned} S_2 = {} & \frac{\hbar}{2\pi}\int_0^{\hbar/T} d\tau \int dx \left[\frac{1}{v_F}(\partial_\tau\phi_{Lc})^2 + v_F(\partial_x\phi_{Lc})^2 + \frac{1}{v_F}(\partial_\tau\phi_{Ls})^2 + v_F(\partial_x\phi_{Ls})^2\right. \\ & \left. + \frac{1}{v_F}(\partial_\tau\phi_{Rc})^2 + v_F(\partial_x\phi_{Rc})^2 + \frac{1}{v_F}(\partial_\tau\phi_{Rs})^2 + v_F(\partial_x\phi_{Rs})^2\right] \\ & + \frac{2E_C}{\pi^2}\int_0^{\hbar/T} d\tau \left[\phi_{Rc}(0,\tau) - \phi_{Lc}(0,\tau) - \frac{\pi N}{\sqrt{2}}\right]^2 \\ & + \frac{2D}{\pi}\int_0^{\hbar/T} d\tau \left\{|r_L|\cos\left[\sqrt{2}\phi_{Lc}(0,\tau)\right]\cos\left[\sqrt{2}\phi_{Ls}(0,\tau)\right]\right. \\ & \left. + |r_R|\cos\left[\sqrt{2}\phi_{Rc}(0,\tau)\right]\cos\left[\sqrt{2}\phi_{Rs}(0,\tau)\right]\right\}, \end{aligned} \tag{A14}$$

$$S_J = \hbar\sum_{\omega_n} J(-i\omega_n)[\phi_{Lc}(i\omega_n) + \phi_{Rc}(i\omega_n)].$$

Up to the order $|r|^2$ the correlation function $\Phi_c(i\omega_n)$ is obtained as

$$\Phi_c(i\omega_n) = \frac{\pi\hbar}{|\omega_n|T} - \frac{\pi}{\hbar\omega_n^2}\sqrt{\gamma T E_C}(|r_L|^2 + |r_R|^2)\int_0^{\hbar/T} \frac{1-\cos\omega_n\tau}{\sin^{3/2}(\pi T\tau/\hbar)}d\tau, \tag{A15}$$

where higher-order terms in T/E_C are neglected. Note that there is no term proportional to $|r_L||r_R|$ in Eq. (A15) because $\langle\cos\phi_{Ls}(0)\rangle = \langle\cos\phi_{Rs}(0)\rangle = 0$. This is an important difference between the two-channel case and four-channel case, implying that the strong-tunneling limit is unstable even on resonance in the latter case. From Eqs. (A7), (A8), (A11), and (A15) we can readily obtain Eq. (45).

3. Asymmetric limit

We have discussed the case of asymmetric barriers in Sec. IV B, where the conductance is calculated in lowest order in v_t but in all orders in $|r_R|$ by effectively summing up all the most divergent terms in perturbation series. Here we shall calculate the leading term in the series, which diverges at low temperatures.

We will calculate the conductance in a way slightly different from the one used in Sec. IV B. We first note that since the left barrier is very high, the left phase fields are pinned at the minima of the potential $\cos(\sqrt{2}\phi_{Lc})\cos(\sqrt{2}\phi_{Ls})$, i.e., $(\phi_{Lc},\phi_{Ls}) = \left(\sqrt{2}\pi(m+\frac{1}{2}), \sqrt{2}\pi n\right)$ or $\left(\sqrt{2}\pi m, \sqrt{2}\pi(n+\frac{1}{2})\right)$, where m and n are integers.[21,38] Thus the tunneling of an electron through the left barrier corresponds to a sudden change in the phase fields, say, $(\phi_{Lc},\phi_{Ls}) = (\pi/\sqrt{2},0) \to (\sqrt{2}\pi,\pi/\sqrt{2})$. The probability for this tunneling process is then given by

$$P_+ = \frac{|\tilde{v}_t|^2}{\hbar^2}\int_{-\infty}^{\infty} \left\langle e^{iH_i t/\hbar} e^{-iH_f t/\hbar}\right\rangle_i e^{ieVt/\hbar} dt, \tag{A16}$$

where $\tilde{v}_t$ is a constant proportional to v_t, V is a voltage across the left barrier, and H_i and H_f are the bosonized Hamiltonian (40)–(42) with the condition $\left(\phi_{Lc}(0),\phi_{Ls}(0)\right) = (\pi/\sqrt{2},0)$ and $(\sqrt{2}\pi,\pi/\sqrt{2})$, respectively. The thermal average $\langle\rangle_i$ is taken for the Hamil-

tonian H_i. It is clear that the probability for the inverse tunneling process is given by $P_- = e^{-eV/T}P_+$. Since the current is $I = 2e(P_+ - P_-)$ with the factor 2 coming from electron spins, the linear conductance is given by

$$G = \frac{2e^2|\tilde{v}_t|^2}{\hbar^2 T}\int_{-\infty}^{\infty}\langle e^{iH_i t/\hbar}e^{-iH_f t/\hbar}\rangle_i dt$$

$$= \frac{2e^2|\tilde{v}_t|^2}{\hbar^2 T}\int_{-\infty}^{\infty}K_0(t)dt. \tag{A17}$$

It will soon become clear that the correlator $K_0(t)$ is related to $K(t)$ in Eq. (52) as $K_0(t) = \{\pi T/iD\sinh[\pi T(t - i\delta)/\hbar]\}^2 K(t)$. For the imaginary time $\tau = it$ we can calculate the correlator using the path integral. To get the conductance we then need to perform the analytic continuation, $\tau \to it + \delta$, and calculate the integral (A17).

The partition function of the system $Z_J^{(2)}|_{J=0}$ is given by Eq. (A13). Since both the charging energy and the cosine potential depend on the phase fields at the point $x = 0$ only, we can integrate out[38] the phase fields off the point to get the following effective action:

$$\begin{aligned} S_{\text{eff}} = {} & \frac{T}{\pi}\sum_{\omega_n}|\omega_n|\left[|\phi_{Lc}(i\omega_n)|^2 + |\phi_{Ls}(i\omega_n)|^2 + |\phi_{Rc}(i\omega_n)|^2 + |\phi_{Rs}(i\omega_n)|^2\right] \\ & + \frac{2E_C}{\pi^2}\int_0^{\hbar/T}d\tau\left[\phi_{Rc}(\tau) - \phi_{Lc}(\tau) - \frac{\pi N}{\sqrt{2}}\right]^2 \\ & + \frac{2D}{\pi}\int_0^{\hbar/T}d\tau\left\{|r_L|\cos\left[\sqrt{2}\phi_{Lc}(\tau)\right]\cos\left[\sqrt{2}\phi_{Ls}(\tau)\right] + |r_R|\cos\left[\sqrt{2}\phi_{Rc}(\tau)\right]\cos\left[\sqrt{2}\phi_{Rs}(\tau)\right]\right\}. \end{aligned} \tag{A18}$$

With this effective action we can readily calculate $K_0(-i\tau)$ as

$$K_0(-i\tau) = \frac{\int\mathcal{D}\phi_{Rc}\int\mathcal{D}\phi_{Rs}\exp\{-\frac{1}{\hbar}S_{\text{eff}}[\phi_{Lc}^{(1)},\phi_{Ls}^{(1)},\phi_{Rc},\phi_{Rs}]\}}{\int\mathcal{D}\phi_{Rc}\int\mathcal{D}\phi_{Rs}\exp\{-\frac{1}{\hbar}S_{\text{eff}}[\phi_{Lc}^{(0)},\phi_{Ls}^{(0)},\phi_{Rc},\phi_{Rs}]\}}, \tag{A19}$$

where

$$\left(\phi_{Lc}^{(0)}(\tau'),\phi_{Ls}^{(0)}(\tau')\right) = \left(\pi/\sqrt{2},0\right), \quad 0 < \tau' < \hbar/T, \tag{A20a}$$

$$\left(\phi_{Lc}^{(1)}(\tau'),\phi_{Ls}^{(1)}(\tau')\right) = \begin{cases}\left(\sqrt{2}\pi,\pi/\sqrt{2}\right), & 0 < \tau' < \tau, \\ \left(\pi/\sqrt{2},0\right), & \tau < \tau' < \hbar/T.\end{cases} \tag{A20b}$$

We shall first consider the noninteracting case, $E_C = 0$, to find the coefficient in Eq. (A17). In this case we may ignore the fields ϕ_{Rc} and ϕ_{Rs}. Thus the correlator is given by

$$K_0(-i\tau) = \frac{\exp\{-\frac{1}{\hbar}S_{\text{eff}}[\phi_{Lc}^{(1)},\phi_{Ls}^{(1)},0,0]\}}{\exp\{-\frac{1}{\hbar}S_{\text{eff}}[\phi_{Lc}^{(2)},\phi_{Ls}^{(2)},0,0]\}} = \left[\frac{\pi T}{D\sin(\pi T\tau/\hbar)}\right]^2, \tag{A21}$$

where we have used Eq. (A6a). Since we know that in this case the conductance is $G = G_L$, we rewrite Eq. (A17) as

$$G = \frac{G_L D^2}{2\pi\hbar T}\int_{-\infty}^{\infty}K_0(t)dt. \tag{A22}$$

With nonzero charging energy, $K_0(-i\tau)$ is calculated up to the order $|r_R|^2$ as

$$K_0(-i\tau) = \frac{\pi^4 T^3}{\gamma E_C D^2}\frac{1}{\sin^3(\pi T\tau/\hbar)}\left\{1 - \frac{\Gamma_R T}{\hbar^2}\int_0^{\tau}d\tau_1\int_{\tau}^{\hbar/T}d\tau_2\frac{1}{\sin[\pi T(\tau_2 - \tau_1)/\hbar]}\right\}, \tag{A23}$$

where Γ_R is defined in Eq. (67). In this calculation we have neglected terms that are smaller in T/E_C such as $T\sum_{\omega_n}\sin(\omega_n\tau_j)[|\omega_n| + (2E_C/\pi\hbar)]^{-1}$. For $0 \le \tau \le \frac{\hbar}{2T}$ the double integral $I_2(\tau)$ is calculated as

$$I_2(\tau) = 2\int_0^{\tau}\frac{\tau_0}{\sin(\pi T\tau_0/\hbar)}d\tau_0 - 2\tau\int_{\tau}^{\hbar/2T}\frac{d\tau_0}{\sin(\pi T\tau_0/\hbar)} = \frac{i\hbar}{T}\left(\frac{\hbar}{2T} - \tau\right) + 2\int_0^{\infty}\frac{t_0}{\sin[\pi T(\tau + it_0)/\hbar]}dt_0. \tag{A24}$$

To get the conductance we need the following integral:

$$\int_{-\infty}^{\infty}\frac{I_2(it)}{\sin^3(i\pi Tt/\hbar)}dt = 2\left(\frac{\hbar}{\pi T}\right)^3\int_{-\infty}^{\infty}dx\int_0^{\infty}dy\frac{y}{\cosh^3 x\cosh(x+y)} = \left(\frac{\hbar}{\pi T}\right)^3[7\zeta(3) - 2]. \tag{A25}$$

From Eqs. (A22), (A23), and (A25) we get

$$G = G_L \frac{\pi^3 T}{8\gamma E_C} \left\{ 1 - \frac{2\Gamma_R}{\pi^3 T} [7\zeta(3) - 2] \right\}. \tag{A26}$$

The leading correction due to the weak reflection in the right contact diverges[14] at low temperatures unless N is a half integer. We note that for $T \gg \Gamma_R$ Eq. (68) in fact reduces to Eq. (A26).

The above result is for the case of electrons with spins. One can perform the same kind of calculation for the spinless case as well. The result has the form

$$G = G_L \frac{2\pi^4 T^2}{3\gamma^2 E_C^2} [1 - \beta |r_R| \cos(2\pi N)], \tag{A27}$$

where β is a numeric coefficient.[39] There are two important differences between Eqs. (A26) and (A27). First, even without the reflection in the right contact, the conductance (A27) is already proportional to T^2. Second, the leading correction gives no additional temperature dependence. Thus the perturbation expansion in powers of $|r_R|$ is well behaved for the spinless case, in striking contrast with the case with spins. A similar difference between the two cases was found for the charge of a quantum dot with a single contact.[15]

APPENDIX B: FOUR-CHANNEL KONDO FIXED POINT APPROACHED FROM THE STRONG COUPLING LIMIT

In this appendix we demonstrate that, when N is a half integer (on resonance), our 1D model is closely related to the four-channel Kondo model by transforming our bosonized Hamiltonian to the one that recently appeared in the study of four-channel Kondo problem.[36]

First we rewrite the Hamiltonian (40)–(42) with the fields introduced in Eq. (A10). We then introduce new fields φ_{Lc}, φ_{Ls}, φ_{Rc}, and φ_{Rs} defined in the same way as φ_+ in Eq. (31), each describing the even modes of ϕ_{Lc}, ϕ_{Ls}, ϕ_{Rc}, and ϕ_{Rs}. The Hamiltonian is now given by

$$H_0 = \frac{v_F}{4\pi} \int \left\{ [\nabla\varphi_{Lc}(x)]^2 + [\nabla\varphi_{Ls}(x)]^2 + [\nabla\varphi_{Rc}(x)]^2 + [\nabla\varphi_{Rs}(x)]^2 \right\} dx, \tag{B1}$$

$$H_C = \frac{E_C}{\pi^2} \left[\varphi_{Rc}(0) - \varphi_{Lc}(0) - \pi N \right]^2, \tag{B2}$$

$$H_V = \frac{2D|r_L|}{\pi} \cos[\varphi_{Lc}(0)] \cos[\varphi_{Ls}(0)] + \frac{2D|r_R|}{\pi} \cos[\varphi_{Rc}(0)] \cos[\varphi_{Rs}(0)]. \tag{B3}$$

With these new fields the current operator is written as

$$I = -\frac{ev_F}{2\pi} \nabla[\varphi_{Lc}(x) + \varphi_{Rc}(x)] \Big|_{x=0}. \tag{B4}$$

Next we fermionize the bosonic fields as in (34) to get

$$H_0 = i\hbar v_F \int \left[\psi_{Lc}^\dagger(x)\nabla\psi_{Lc}(x) + \psi_{Ls}^\dagger(x)\nabla\psi_{Ls}(x) + \psi_{Rc}^\dagger(x)\nabla\psi_{Rc}(x) + \psi_{Rs}^\dagger(x)\nabla\psi_{Rs}(x) \right] dx, \tag{B5}$$

$$H_C = E_C \left\{ \int \mathrm{sgn}(x) \left[\psi_{Lc}^\dagger(x)\psi_{Lc}(x) - \psi_{Rc}^\dagger(x)\psi_{Rc}(x) \right] dx - N' \right\}^2, \tag{B6}$$

$$\begin{aligned} H_V = & -\hbar v_F |r_L| \eta_{Lc}\eta_{Ls} \left[\psi_{Lc}(0) - \psi_{Lc}^\dagger(0) \right] \left[\psi_{Ls}(0) - \psi_{Ls}^\dagger(0) \right] \\ & -\hbar v_F |r_R| \eta_{Rc}\eta_{Rs} \left[\psi_{Rc}(0) - \psi_{Rc}^\dagger(0) \right] \left[\psi_{Rs}(0) - \psi_{Rs}^\dagger(0) \right], \end{aligned} \tag{B7}$$

$$I = -ev_F : \psi_{Lc}^\dagger(0)\psi_{Lc}(0) + \psi_{Rc}^\dagger(0)\psi_{Rc}(0) :, \tag{B8}$$

where the η's are Majorana fermions, the ψ's chiral fermions, and $N' = N + \text{const}$. Since the products of the Majorana fermions, $\eta_{Lc}\eta_{Ls}$ and $\eta_{Rc}\eta_{Rs}$, commute with each other, and $(\eta_{Lc}\eta_{Ls})^2 = (\eta_{Rc}\eta_{Rs})^2 = -1$, we can replace them by $-i$. We then introduce another set of fermions:

$$\psi_{1c}(x) = \frac{1}{\sqrt{2}} \left[\psi_{Lc}(x) + i\psi_{Rc}^\dagger(x) \right], \tag{B9a}$$

$$\psi_{2c}(x) = \frac{1}{\sqrt{2}} \left[\psi_{Lc}^\dagger(x) + i\psi_{Rc}(x) \right], \tag{B9b}$$

$$\psi_s(x) = \frac{1}{2} \left[\psi_{Ls}(x) - \psi_{Ls}^\dagger(x) + i\psi_{Rs}(x) - i\psi_{Rs}^\dagger(x) \right], \tag{B9c}$$

$$\tilde{\psi}_s(x) = \frac{1}{2}\left[\psi_{Ls}(x) + \psi^\dagger_{Ls}(x) + i\psi_{Rs}(x) + i\psi^\dagger_{Rs}(x)\right]. \tag{B9d}$$

In terms of these fermions the Hamiltonian and the current operator are written as

$$H_0 = i\hbar v_F \int \left[\psi^\dagger_{1c}(x)\nabla\psi_{1c}(x) + \psi^\dagger_{2c}(x)\nabla\psi_{2c}(x) + \psi^\dagger_s(x)\nabla\psi_s(x) + \tilde{\psi}^\dagger_s(x)\nabla\tilde{\psi}_s(x)\right] dx, \tag{B10}$$

$$H_C = E_C \left\{\int \mathrm{sgn}(x)\left[\psi^\dagger_{1c}(x)\psi_{1c}(x) - \psi^\dagger_{2c}(x)\psi_{2c}(x)\right] dx - N'\right\}^2, \tag{B11}$$

$$\begin{aligned} H_V = {} & \frac{i}{\sqrt{2}}\hbar v_F |r_L| \left[\psi_{1c}(0) - \psi^\dagger_{1c}(0) - \psi_{2c}(0) + \psi^\dagger_{2c}(0)\right]\left[\psi_s(0) - \psi^\dagger_s(0)\right] \\ & + \frac{i}{\sqrt{2}}\hbar v_F |r_R| \left[\psi_{1c}(0) + \psi^\dagger_{1c}(0) - \psi_{2c}(0) - \psi^\dagger_{2c}(0)\right]\left[\psi_s(0) + \psi^\dagger_s(0)\right], \end{aligned} \tag{B12}$$

$$I = -ev_F\left[\psi^\dagger_{1c}(0)\psi^\dagger_{2c}(0) + \psi_{2c}(0)\psi_{1c}(0)\right]. \tag{B13}$$

The fermion field $\tilde{\psi}_s$ is decoupled from the other fields, and thus we neglect it in the following. We bosonize the remaining three fermions again:

$$\psi_{1c}(x) = \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2} \eta_{1c} e^{i\varphi_1(x)}, \tag{B14a}$$

$$\psi_{2c}(x) = \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2} \eta_{2c} e^{i\varphi_2(x)}, \tag{B14b}$$

$$\psi_s(x) = \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2} \eta_s e^{i\varphi_s(x)}, \tag{B14c}$$

where the φ's are bosonic fields and the η's are Majorana fermions. Equations (B10)–(B13) are then rewritten as

$$H_0 = \frac{v_F}{4\pi}\int \left\{[\nabla\varphi_1(x)]^2 + [\nabla\varphi_2(x)]^2 + [\nabla\varphi_s(x)]^2\right\} dx, \tag{B15}$$

$$H_C = \frac{E_C}{\pi^2}\left[\varphi_2(0) - \varphi_1(0) - \pi\left(N - \frac{1}{2}\right)\right]^2, \tag{B16}$$

$$\begin{aligned} H_V = {} & -\frac{\sqrt{2}}{\pi} D|r_L| \left[\eta_1 \sin\varphi_1(0) - \eta_2 \sin\varphi_2(0)\right] \sin\varphi_s(0) \\ & + \frac{\sqrt{2}}{\pi} D|r_R| \left[\eta_1 \cos\varphi_1(0) - \eta_2 \cos\varphi_2(0)\right] \cos\varphi_s(0), \end{aligned} \tag{B17}$$

$$I = -i\frac{eD}{\hbar}\eta_1\eta_2 \sin[\varphi_1(0) + \varphi_2(0)], \tag{B18}$$

where $\eta_1 = i\eta_{1c}\eta_s$ and $\eta_2 = i\eta_{2c}\eta_s$. In Eq. (B16) the parameter N is shifted by $\frac{1}{2}$ so that the Hamiltonian (B15)–(B17) yields the same perturbation series for the partition function in powers of $|r|$ as the original Hamiltonian (B1)–(B3).

Let us consider the case where N is a half integer (on resonance) and $|r_L| = |r_R| = r_0$ (symmetric barriers). At low temperatures the difference $\varphi_2(0) - \varphi_1(0)$ is almost fixed to be $\pi(N - \frac{1}{2})$ due to the charging energy. In the strong-tunneling limit ($r_0 \ll 1$), we can thus take an average of the Hamiltonian over $\varphi_2 - \varphi_1$ to get an effective Hamiltonian for φ_s and $\bar{\varphi} = (\varphi_1 + \varphi_2)/\sqrt{2}$:

$$\langle H_0 + H_V\rangle = \frac{v_F}{4\pi}\int \left\{[\nabla\bar{\varphi}(x)]^2 + [\nabla\varphi_s(x)]^2\right\} dx + \frac{2}{\pi}\left(\frac{\gamma E_C}{\pi D}\right)^{1/4} D r_0(\eta_1 - \eta_2)\cos\left(\frac{\bar{\varphi}(0)}{\sqrt{2}} + \varphi_s(0)\right), \tag{B19}$$

$$I = -i\frac{eD}{\hbar}\eta_1\eta_2 \sin\left[\sqrt{2}\,\bar{\varphi}(0)\right]. \tag{B20}$$

Introducing new bosonic fields, $\varphi_a = \sqrt{\frac{2}{3}}\bar{\varphi} - \sqrt{\frac{1}{3}}\varphi_s$ and $\varphi_b = \sqrt{\frac{1}{3}}\bar{\varphi} + \sqrt{\frac{2}{3}}\varphi_s$, we rewrite the Hamiltonian and the current as

$$\langle H_0 + H_V\rangle = \frac{v_F}{4\pi}\int \left\{[\nabla\varphi_a(x)]^2 + [\nabla\varphi_b(x)]^2\right\} dx + \frac{2\sqrt{2}}{\pi}\left(\frac{\gamma E_C}{\pi D}\right)^{1/4} D r_0 \sigma_x \cos\left[\sqrt{\frac{3}{2}}\varphi_b(0)\right], \tag{B21}$$

$$I = \frac{eD}{\hbar}\sigma_z \sin\left[\frac{2}{\sqrt{3}}\varphi_a(0) + \sqrt{\frac{2}{3}}\varphi_b(0)\right], \tag{B22}$$

where $\sigma_x = (\eta_1 - \eta_2)/\sqrt{2}$ and $\sigma_z = -i\eta_1\eta_2$ are Pauli matrices. The Hamiltonian (B21) is the same as that of a single impurity in the $g = \frac{3}{4}$ Luttinger liquid.[23] Since this $g = \frac{3}{4}$ problem is related to the four-channel Kondo problem in the Toulouse limit,[36] the above transformations show that in the strong-tunneling limit our bosonized Hamiltonian is indeed equivalent to the four-channel Kondo problem. The infrared stable fixed point of the four-channel Kondo problem corresponds to the case with $r_0 \to \infty$ in Eq. (B21). At this point $\sigma_x \cos[(3/2)^{1/2}\varphi_b]$ takes -1 so that the field $\varphi_b(0)$ is pinned at either $\varphi_b(0) = \pm 2n(2/3)^{1/2}\pi$ (for $\sigma_x = -1$) or $\varphi_b(0) - \pi = \pm 2m(2/3)^{1/2}\pi$ (for $\sigma_x = 1$), where m and n are integers. Thus in calculating the dimension of the current operator (B22) we may neglect $\varphi_b(0)$. We then immediately see that the dimension is $d = 1$ because the operator $\exp[i(4/3)^{1/2}\varphi_a(0)]$ has dimension 2/3 and the dimension of σ_z is[36] 1/3. This means that the leading term of the conductance is temperature-independent at $T \to 0$. To calculate the conductance we need to know the coefficient λ of the correlator $\langle [I(t), I(0)] \rangle = -\lambda e^2/t^2$ at $T \to 0$, which is a difficult problem. We expect, however, that λ should be universal, implying that at $T = 0$ the linear conductance G has a universal value $\pi\lambda e^2/2\hbar$ independent of the initial value of r_0.

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[1] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).

[2] *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992).

[3] M. A. Kastner, Rev. Mod. Phys. **64**, 849 (1992).

[4] L. I. Glazman and R. I. Shekhter, J. Phys. Condens. Matter **1**, 5811 (1989).

[5] D. V. Averin and Yu. V. Nazarov, Phys. Rev. Lett. **65**, 2446 (1990); D. V. Averin and A. A. Odintsov, Phys. Lett. A **140**, 251 (1989).

[6] N. C. van der Vaart, A. T. Johnson, L. P. Kouwenhoven, D. J. Maas, W. de Jong, M. P. de Ruyter van Steveninck, A. van der Enden, C. J. P. M. Harmans, and C. T. Foxon, Physica (Amsterdam) **189B**, 99 (1993).

[7] C. Pasquier, U. Meirav, F. I. B. Williams, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **70**, 69 (1993).

[8] F. R. Waugh, M. J. Berry, D. J. Mar, R. M. Westervelt, K. C. Campman, and A. C. Gossard, Phys. Rev. Lett. **75**, 705 (1995).

[9] L. W. Molenkamp, K. Flensberg, and M. Kemerink, Phys. Rev. Lett. (to be published).

[10] F. Guinea and G. Schön, J. Low Temp. Phys. **69**, 219 (1987).

[11] S. V. Panyukov and A. D. Zaikin, Phys. Rev. Lett. **67**, 3168 (1991).

[12] G. Falci, G. Schön, and G. T. Zimanyi, Phys. Rev. Lett. **74**, 3257 (1995).

[13] H. Schoeller and G. Schön, Phys. Rev. B **50**, 18 436 (1994).

[14] K. Flensberg, Phys. Rev. B **48**, 11 156 (1993); Physica B **203**, 432 (1994).

[15] K. A. Matveev, Phys. Rev. B **51**, 1743 (1995).

[16] L. I. Glazman and K. A. Matveev, Zh. Eksp. Teor. Fiz. **98**, 1834 (1990) [Sov. Phys. JETP **71**, 1031 (1990)].

[17] K. A. Matveev, Zh. Eksp. Teor. Fiz. **99**, 1598 (1991) [Sov. Phys. JETP **72**, 892 (1991)].

[18] P. W. Anderson, J. Phys. C **3**, 2436 (1970).

[19] L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1991)].

[20] J. Sólyom, Adv. Phys. **28**, 201 (1979); V. J. Emery, in *Highly Conducting One-Dimensional Solids*, edited by J. Devreese *et al.* (Plenum, New York, 1979); H. Fukuyama and H. Takayama, in *Electronic Properties of Inorganic Quasi-One-Dimensional Materials,* edited by P. Monceau (Reidel, Dordrecht, 1985); H. J. Schulz, in *Correlated Electron Systems,* edited by V. J. Emery (World Scientific, Singapore, 1993).

[21] A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 3827 (1993).

[22] The spinless case discussed in Sec. III corresponds to the case $\eta = 1$ and $\Delta\epsilon \ll T \ll U$ in Ref. 21.

[23] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15 233 (1992).

[24] F. Guinea, Phys. Rev. B **32**, 7518 (1985).

[25] The idea of refermionization goes back to A. Luther and V. J. Emery, Phys. Rev. Lett. **33**, 589 (1974).

[26] Strictly speaking, the presence of the Majorana fermion η_+ in Eq. (34) is not necessary, since we have only one bosonic branch. Nevertheless, it is more convenient to add η_+, so that the resulting Hamiltonian (35) had only terms quadratic in fermion operators.

[27] P. Nozières and A. Blandin, J. Phys. (Paris) **41**, 193 (1980).

[28] See, e.g., the review by G.-L. Ingold and Yu. V. Nazarov in *Single Charge Tunneling* (Ref. 2).

[29] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).

[30] K. Schotte and U. Schotte, Phys. Rev. **182**, 479 (1969).

[31] P. Nozières and C. T. de Dominicis, Phys. Rev. **178**, 1097 (1969).

[32] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1991).

[33] A formal proof can be obtained in a way similar to the derivation (54)–(58) of the expression $K(t) = K_\Theta(t)$ for the correlator (53). Instead of the transformation (54) one should use $U = (-1)^{\hat{n}c^\dagger c}$.

[34] A. M. Tsvelik and P. B. Wiegmann, Z. Phys. B **54**, 201, (1984); N. Andrei and C. Destri, Phys. Rev. Lett. **52**, 364 (1984).

[35] I. Affleck and A. W. W. Ludwig, Nucl. Phys. **B360**, 641 (1991).

[36] M. Fabrizio and A. O. Gogolin, Phys. Rev. B **50**, 17732 (1994).

[37] A. I. Larkin and V. I. Melnikov, Zh. Eksp. Teor. Fiz. **61**, 1231 (1971) [Sov. Phys. JETP **34**, 656 (1972)].

[38] A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 4631 (1993).

[39] H. Yi and C. L. Kane (unpublished).