Coulomb blockade of activated conduction

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We present a theory of the Coulomb-blockade oscillations of conductance through a quantum dot at relatively high temperatures, when the conduction is achieved via thermal activation of electrons over the tunnel barriers. We discover that the oscillations of conductance as a function of the external gate voltage persist in this regime, and find their shape. [S0163-1829(96)02340-5]

Recent progress in microfabrication technology allowed the creation of nanostructures with controllable conductance. The transport of electrons in such devices is strongly affected by the Coulomb interactions. The most striking manifestation of these interactions is the phenomenon of the Coulomb blockade of conductance between two leads coupled to a small conducting region by tunnel barriers. This phenomenon is observed in both metallic devices[1] and GaAs heterostructures[2] at temperatures of the order of 1 K. The Coulomb blockade in such structures manifests itself as periodic peaks in conductance as a function of the gate voltage, and is due to the discreteness of the charge transferred in each tunneling event.

The latest technological development has lead to the creation of silicon-based Coulomb blockade devices with working temperatures of order 100 K and above.[3,4] At such high temperatures one can expect the transport through the barriers to be dominated by the thermal activation of electrons over the barriers rather than tunneling through them. In the present paper we study the Coulomb blockade oscillations of conductance in this activation regime.

We start with a discussion of the conditions at which the transport through a barrier is dominated by thermal activation. The conductance G of a tunnel junction at arbitrary temperature T is given by the following expression:

$$G_b=\frac{2e^2}{h}\int_{-\infty}^{+\infty}\left[-\frac{\partial n_F(\epsilon)}{\partial\epsilon}\right]\mathcal{T}(\epsilon)\,d\epsilon, \tag{1}$$

where $\partial n_F/\partial\epsilon=-[4T\cosh^2(\epsilon/2T)]^{-1}$ is the derivative of the Fermi function, and h is the Planck's constant. In Eq. (1) we assumed that the transport is essentially one dimensional, which is usually a good approximation for barriers formed by constricting the electron gas in the transverse direction.[5]

In the simplest case of a parabolic potential barrier $U(x)=U-\frac{1}{2}m\omega^2x^2$, the transmission coefficient is[6]

$$\mathcal{T}(\epsilon)=\frac{1}{1+\exp\left(\dfrac{U-\epsilon}{T_0}\right)},\qquad T_0=\frac{h\omega}{(2\pi)^2}. \tag{2}$$

The energy interval making the dominant contribution to integral (1) depends on the relation between T and T_0. At $T<T_0$ the region of ϵ near zero dominates, i.e., the conductance is determined by the tunneling of electrons with energies near the Fermi level. On the other hand, at higher temperatures, $T>T_0$, the main contribution to integral (1) comes from energies $\epsilon\approx U$, meaning that the activated transport dominates the conduction.

For reasonably long barriers (e.g., a barrier formed by the gate-induced depletion) the parabolic approximation of the barrier shape works well for all the energies from the top of the barrier down to the Fermi level, provided the zero-temperature conductance is not anomalously small.[5] Moreover, the threshold temperature T_0 above which activation becomes the dominant mechanism of electron transport exists for a barrier of arbitrary shape. Thus the following results are insensitive to this approximation.

In the following we will be considering temperatures exceeding T_0. In this activated regime the conductance (1) is easily found

$$G_b(U)=\frac{2e^2}{h}\,\frac{\pi T_0}{T\,\sin(\pi T_0/T)}\,e^{-U/T},\qquad U\gg T>T_0. \tag{3}$$

Let us now consider the transport between two leads coupled to a small conductor (quantum dot) by two barriers, Fig. 1. In a generic case the heights of the two barriers are different, and the conductance of one of the contacts (say, the right one) is much larger than that of the other one. In this case the quantum dot is in equilibrium with the right lead, and its thermodynamics is completely described by the partition function

$$Z(N)=\sum_n e^{-E_n(N)/T}. \tag{4}$$

Here $E_n(N)=E_C(n-N)^2$ is the electrostatic energy of the system, which depends on the total capacitance C of the dot with respect to all the electrodes, $E_C=e^2/2C$; parameter $N=C_gV_g/e$ is proportional to the voltage V_g applied to a gate electrode and to the capacitance C_g of the dot with respect to the gate; n is the number of electrons in the dot counted from its equilibrium value at $V_g=0$.

The conductance of the system is determined by the transmission of electrons through the left barrier. In the tunneling

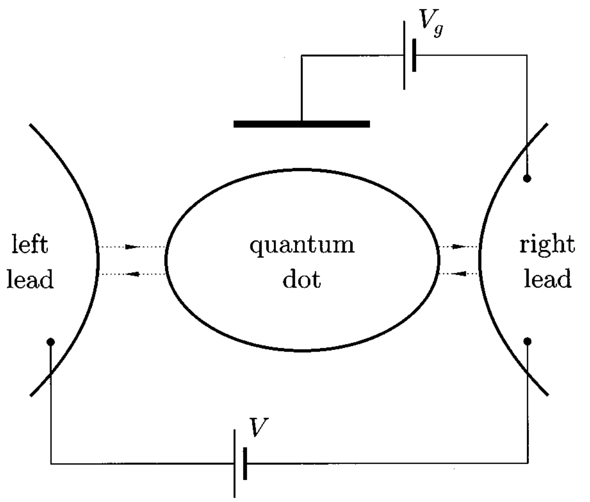

FIG. 1. Schematic view of the quantum-dot system. The applied bias V which drives the current, is small, $eV \ll T$. Linear conductance $G(N)$ [see Eq. (9)] is controlled by the gate voltage $V_g \equiv eN/C_g$.

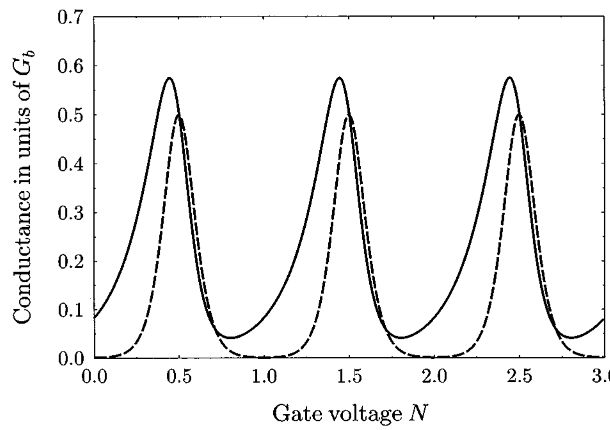

FIG. 2. Peaks (10) in conductance as a function of gate voltage for the cases $\lambda = 0.25$ and $T = 0.1E_C$ (solid line). The peaks are asymmetric and wider than those for the conventional Coulomb blockade (Ref. 7), $G(N) = G_b(E_C\Delta N/T)/\sinh(2E_C\Delta N/T)$, at the same temperature (dashed line).

regime, the rate of electron transitions between the left lead and the dot is determined by the transmission coefficient of the barrier and by the occupation probabilities of the corresponding electronic states. The latter give rise to the nontrivial dependence of the conductance on the gate voltage.[7] In the activated regime, the current through the barrier is dominated by the electrons with high energies $\epsilon \approx U$, at which the occupation numbers are exponentially small. (We assume that the barrier height is large compared to the charging energy, $U \gg E_C$.) Thus the transmission through the barrier is determined exclusively by its height measured from the Fermi level. If the bias voltage V is applied to the leads, we find the current through the dot as

$$I = e\sum_n \left[w_n R(U_n - eV) - w_{n+1} R(U_n - E_{n+1} + E_n) \right]. \tag{5}$$

Here $w_n = e^{-E_n/T}/Z$ is the probability of the dot to have charge en, and $R(U)$ is the rate of activated transfer of electrons over the barrier of height U,

$$R(U) = \frac{2}{h}\int_{-\infty}^{\infty} n_F(\epsilon)\mathcal{T}(\epsilon)\,d\epsilon = \frac{2\pi T_0}{h\,\sin(\pi T_0/T)}e^{-U/T}. \tag{6}$$

In the last term in Eq. (5) we accounted for the fact that the Fermi level for the $(n+1)$st electron is shifted by $E_{n+1} - E_n$ due to the charging energy. The substitution of Eq. (6) into Eq. (5) gives the following result for the linear conductance:

$$G = \frac{2e^2}{h}\,\frac{\pi T_0}{T\,\sin(\pi T_0/T)}\sum_n w_n e^{-U_n/T}. \tag{7}$$

In Eqs. (5) and (7) we allowed the dependence of the height U of the left barrier on n, which may be caused by the effect of the electric field of the charged dot. If one neglects this dependence, $U_n = U$, the conductance (7) of the system coincides with the conductance (3) of the left barrier, $G_b(U)$, and is independent of the gate voltage. In this case no Coulomb blockade oscillations would be observed.

We now show that the dependence of the barrier height on the charge of the dot inevitably restores the oscillations of conductance $G(N)$. Indeed, when an electron moves from the left lead into the dot with n electrons, it overcomes the barrier U_n affected by the electric field of the charged dot. The dependence of the barrier height on the charge of the dot can be found as follows. The chemical potential for the $(n+1)$st electron in the dot is shifted due to the charging effects by $E_{n+1} - E_n$. This shift is due to the Coulomb interaction of the $(n+1)$st electron with all the other electrons in the dot, and can be interpreted as a voltage $V_n = (E_{n+1} - E_n)/e$ applied to the dot. As a result, the potential barrier is increased by some fraction λ of this voltage,

$$U_n = U + \lambda(E_{n+1} - E_n). \tag{8}$$

Here the constant λ is determined by geometry of the contact, and $0 < \lambda < 1$; for a symmetric contact we expect $\lambda = \frac{1}{2}$. We now substitute Eq. (8) into Eq. (7), and find

$$G(N) = G_b(U)\,\frac{Z(N-\lambda)}{Z(N)}\,e^{-\lambda(1-\lambda)E_C/T}, \tag{9}$$

where $G_b(U)$ is the conductance (3) of the left barrier.

At nonzero λ the conductance (9) does depend on the (dimensionless) gate voltage N. It is instructive to compare the shapes of the conductance peaks $G(N)$ given by Eq. (9) with those in the case of conventional Coulomb blockade.[7] Assuming that the charging energy E_C exceeds T_0, one can choose the temperature T in the interval $T_0 < T \ll E_C$. In this case the Coulomb blockade of the activated conductance manifests itself as narrow asymmetric peaks,

$$G(N) = G_b(U)\,\frac{e^{E_C(2\lambda-1)\Delta N/T}}{e^{E_C\Delta N/T} + e^{-E_C\Delta N/T}}; \tag{10}$$

see Fig. 2. Here ΔN is the dimensionless gate voltage N measured from the nearest half-integer value. The centers of the peaks are shifted from their positions in the case of conventional Coulomb blockade by $\Delta N=(T/2E_C)\ln[\lambda/(1-\lambda)]$. The asymmetry disappears only in the case $\lambda=\frac{1}{2}$, and then the peaks are qualitatively similar to those[7] for the blockade of tunneling conductance. It is interesting, however, that the activation energy $E_C\Delta N$ for the conductance (10) at off-peak values of the gate voltage is smaller than that of Ref. 7 by a factor of 2.

At higher temperatures, $T\gtrsim E_C$, the conductance peaks transform into weak oscillations,

$$G(N)=G_b(U)[1+4e^{-\pi^2 T/E_C}\sin\pi\lambda\ \sin(2\pi N-\pi\lambda)]. \tag{11}$$

The shift of the maxima in conductance from their normal positions $N=(2n+1)/2$ is now more pronounced: $\Delta N=\frac{1}{2}(\lambda-\frac{1}{2})$.

In conclusion, we have shown that the Coulomb blockade phenomenon does not require the *tunneling* mechanism of conduction through the contacts separating the quantum dot from leads. It exists in the regime of the *activation* conduction as well,[8] due to a periodic modulation in the gate voltage of the barriers' heights. This periodic modulation is a direct result of the charge quantization. The charge of the dot is quantized, and therefore the Coulomb blockade exists, as long as the resistance of the contacts exceeds[1] h/e^2. However, the shape of the peaks in the conductance vs gate voltage dependence does reflect the dominating mechanism. For the activation transport, the peaks generally are asymmetric, and are shifting with the increase of temperature.

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