Asymmetric Zero-Bias Anomaly for Strongly Interacting Electrons in One Dimension

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We study a system of one-dimensional electrons in the regime of strong repulsive interactions, where the spin exchange coupling J is small compared with the Fermi energy, and the conventional Tomonaga-Luttinger theory does not apply. We show that the tunneling density of states has a form of an asymmetric peak centered near the Fermi level. In the spin-incoherent regime, where the temperature is large compared to J, the density of states falls off as a power law of energy ε measured from the Fermi level, with the prefactor at positive energies being twice as large as that at the negative ones. In contrast, at temperatures below J the density of states forms a split peak with most of the weight shifted to negative ε.

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The discovery of conductance quantization in quantum wires [1] has stimulated interest in transport properties of one-dimensional conductors. From the theoretical point of view, these systems are interesting because in one dimension interacting electrons form the so-called Luttinger liquid [2], with properties very different from the conventional Fermi liquids. A number of nontrivial properties of the Luttinger liquid, such as the power-law dependence of the tunneling density of states on energy and temperature, have been recently observed experimentally [3–5].

It is important to note that the Luttinger-liquid picture [2] describes only the low-energy properties of the system. Quantitatively, this means that all the important energy scales, such as the temperature T, must be much smaller than the typical bandwidth of the problem. In a system of spin-$\frac{1}{2}$ electrons the charge and spin excitations propagate at different velocities [6], resulting in two bandwidth parameters. In the noninteracting case both bandwidths are equal to the Fermi energy $E_F = (\pi\hbar n)^2/8m$, where n is the electron density and m is the effective mass. Repulsive interactions between electrons increase the charge bandwidth D_ρ and decrease the spin bandwidth D_σ. At moderate interaction strength both bandwidths remain of the order of Fermi energy E_F, and the Luttinger-liquid picture applies at $T \ll E_F$. On the other hand, if the interactions are strong, the exchange coupling of electron spins J is strongly suppressed, and $D_\sigma \sim J \ll D_\rho$. As a result the Luttinger-liquid picture applies only at $T \ll J$, and there appears an interesting new regime when the temperature is in the range $J \ll T \ll D_\rho$. In this regime the temperature does not strongly affect the charge excitations in the system, but completely destroys the ordering of the electron spins.

A number of interesting phenomena have been predicted in this so-called *spin-incoherent regime*. The destruction of spin order may be responsible [7] for the anomalous quantization of conductance in the experiments [8]. Furthermore, contrary to the conventional Luttinger-liquid picture, the tunneling density of states may show a power-law *peak* near the Fermi level [9,10] even in the case of repulsive interactions,

$$\nu(\varepsilon) \propto \frac{|\varepsilon|^{(1/4K_\rho)-1}}{\sqrt{\ln(D_\rho/|\varepsilon|)}}, \qquad J \ll T \ll |\varepsilon| \ll D_\rho. \qquad (1)$$

Here ε is the energy of the tunneling electron measured from the Fermi level. The result (1) was first obtained [9] for the Hubbard model in the limit of strong on-site repulsion $U \to \infty$. In this case the interactions have a very short range, resulting in $K_\rho = 1/2$ and $\nu \propto [|\varepsilon| \ln(D_\rho/|\varepsilon|)]^{-1/2}$. For longer-range interactions K_ρ is below $1/2$, but as long as $K_\rho > 1/4$, the density of states has a peak at low energies.

A similar enhancement of the density of states at $\varepsilon \ll D_\rho$ was predicted earlier [11] in the strongly interacting limit of the Hubbard model at zero temperature,

$$\nu(\varepsilon) \propto |\varepsilon|^{-3/8}, \qquad T = 0, \qquad J \ll |\varepsilon| \ll D_\rho. \qquad (2)$$

Here the effective exchange coupling of electron spins $J \sim t^2/U$, the bandwidth $D_\rho \sim t$, and t is the hopping matrix element in the Hubbard model. The different power-law behaviors of the density of states (1) and (2) point to the nontrivial physics developing when the temperature T is lowered below the exchange J, even if they are both small compared to the energy ε.

In this Letter we develop a unifying theory, which enables one, in principle, to obtain the density of states in a system of strongly interacting one-dimensional electrons at arbitrary ratio T/J, as long as T, $J \ll \varepsilon$. In the spin-incoherent case, $T \gg J$, our theory reproduces the result (1). Furthermore, we show that true asymptotic behavior of $\nu(\varepsilon)$ at low energies is given by Eq. (1) even at $T = 0$. Most importantly, in both cases the peak in $\nu(\varepsilon)$ is very asymmetric. In particular, the 3/8-power law (2) appears for short-range interactions at moderately low positive energies ε, but never at $\varepsilon < 0$.

Our approach is based on the fact that at strong interactions, when $J/D_\rho \to 0$, the Hamiltonian of the system of one-dimensional electrons can be written as a sum of two contributions describing the charge and spin degrees of freedom, $H = H_\rho + H_\sigma$, with

$$H_\sigma = \sum_l J \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}. \tag{3}$$

This result was first obtained by Ogata and Shiba [12] in the $U/t \to \infty$ limit of the Hubbard model. In the case of quantum wires at low electron densities a similar decoupling of charge and spin degrees of freedom was discussed in Ref. [7]. This decoupling occurs whenever the repulsive interactions are so strong that even electrons with opposite spins do not occupy the same point in space. In this case the electrons are well separated from each other, and their spins form a Heisenberg spin chain (3). Strong repulsion between electrons suppresses the exchange of nearest-neighbor spins, $J \ll E_F$; the coupling of next-nearest neighbors is negligible. Thus Eq. (3) is a universal spin Hamiltonian for one-dimensional electron systems with strong repulsion, such as the Hubbard model [12] at $U \gg t$ and quantum wires at low electron densities [7].

Since the Pauli principle is effectively enforced by the interactions even for electrons with opposite spins, the charge part H_ρ of the Hamiltonian can be written in terms of spinless fermions (holons). In the case of the Hubbard model [12] the holons are noninteracting, because two holons never occupy the same lattice site, and the interaction range does not extend beyond single site. On the other hand, if the original interaction between electrons has nonzero range, the holons do interact. Since we are only interested in the density of states at energies ε low compared to the bandwidth D_ρ, the holon Hamiltonian can be bosonized,

$$H_\rho = \int \frac{\hbar u_\rho}{2\pi}[K(\partial_x\theta)^2 + K^{-1}(\partial_x\phi)^2]dx. \tag{4}$$

Here u_ρ is the speed of the charge excitations and K is related to the standard Luttinger-liquid parameter K_ρ for the charge modes in an interacting electron system [2] as $K = 2K_\rho$, with the factor of 2 originating from the different definition of the bosonic fields ϕ and θ. In the limit $U/t \to \infty$ of the Hubbard model the holons do not interact, and $K = 1$.

To find the tunneling density of states, one needs an expression for the electron creation and annihilation operators. The electron annihilation operator $\psi_\sigma(x)$ affects both the charge and spin degrees of freedom: it destroys a holon at point x and removes a site with spin σ from the spin chain (3). Building on the ideas of Refs. [7,10,11,13] we write the electron annihilation operator as

$$\psi_\sigma^R(x) = \frac{e^{i[k_F x + \phi(x) - \theta(x)]}}{(2\pi\alpha)^{1/2}} Z_{l,\sigma}|_{l=[k_F x+\phi(x)]/\pi}. \tag{5}$$

The first factor is the bosonized form of an operator destroying a right-moving holon; the full expression for $\psi_\sigma(x)$ consists of the term (5) and a similar expression for the left-moving branch. (Here $\alpha = \hbar u_\rho/D_\rho$ is the short-distance cutoff; the holon Fermi momentum is related to the mean electron density n as $k_F = \pi n$.) The operator $Z_{l,\sigma}$ introduced in Ref. [11] removes site l with spin σ from the spin chain. It is important to note that Eq. (5) properly accounts for the fact [7,10] that charge modes shift the spin chain at point x by $\delta l = \phi(x)/\pi$ with respect to its average position $\bar{l} = k_F x/\pi$.

It is convenient to express the operators $Z_{l,\sigma}$ in terms of their Fourier transforms $z_\sigma(q)$, where the momentum q is defined on a lattice and assumes values between $-\pi$ and π. Then the operator (5) takes the form

$$\psi_\sigma^R(x) = \int_{-\pi}^{\pi} \frac{dq}{2\pi} z_\sigma(q) \frac{e^{ik_F[1+(q/\pi)]x}}{(2\pi\alpha)^{1/2}} e^{i(1+\frac{q}{\pi})\phi(x) - i\theta(x)}. \tag{6}$$

In addition to the holon destruction operator $e^{i(\phi-\theta)}$ the integrand contains a factor $e^{i(q/\pi)\phi}$. Similar factors appear when bosonization is applied to the problem of x-ray-edge singularity [14,15], where they represent the effect of the core-hole potential on the electronic wave functions. More specifically, a core hole with the scattering phase shift δ adds a factor $e^{i(2\delta/\pi)\phi}$ to the fermion operator. Thus according to Eq. (6) the electron tunneling process that changes the momentum of the spin chain by q also adds a scattering phase shift for the holons $\delta = q/2$. This observation is consistent with the fact [11] that for the state of momentum Q of the spin chain, the periodic boundary conditions for the holons acquire a phase factor e^{iQ}.

The tunneling density of states $\nu(\varepsilon)$ can be computed as imaginary part of the electron Green's function. At $|\varepsilon| \gg J$ one can neglect the time dependence of the operators $z_\sigma(q)$ in Eq. (6) and use their static correlation functions. Utilizing the standard techniques [2] to average exponentials of time-dependent bosonic fields ϕ and θ in Eq. (6), in the limit of zero temperature we obtain

$$\nu_\sigma^\pm(\varepsilon) = \nu_0 \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{c_\sigma^\pm(q)}{\Gamma(\lambda(q)+1)}\left(\frac{|\varepsilon|}{D_\rho}\right)^{\lambda(q)} \tag{7}$$

for positive and negative ε. Here $\nu_0 = (\pi\hbar u_\rho)^{-1}$ and the exponent $\lambda(q)$ is given by

$$\lambda(q) = \frac{1}{2}\left[\left(1+\frac{q}{\pi}\right)^2 K + \frac{1}{K}\right] - 1. \tag{8}$$

We have also defined the static correlation functions

$$c_\sigma^+(q) = \sum_l \langle Z_{l,\sigma} Z_{0,\sigma}^\dagger\rangle e^{-iql}, \tag{9a}$$

$$c_\sigma^-(q) = \sum_l \langle Z_{0,\sigma}^\dagger Z_{l,\sigma}\rangle e^{-iql}, \tag{9b}$$

with averaging performed over the ground state of the spin chain (3).

At $|\varepsilon| \ll D_\rho$ the dominant contribution to the integral in Eq. (7) comes from the vicinity of its lower limit, $q = -\pi$.

The $|\varepsilon|/D_\rho \to 0$ asymptote has the form

$$\nu_\sigma^\pm(\varepsilon) = \nu_0 \sqrt{\frac{\pi}{8K}} \frac{c_\sigma^\pm(\pi)}{\Gamma(\frac{1}{2K})} \left(\frac{|\varepsilon|}{D_\rho}\right)^{(1/2K)-1} \frac{1}{\sqrt{\ln\frac{D_\rho}{|\varepsilon|}}}. \quad (10)$$

This result assumes that the functions (9) do not vanish at $q = \pm\pi$.

It is important to point out that even though we so far assumed $T = 0$, the low-energy asymptote (10) agrees with Eq. (1), rather than Eq. (2). Indeed, in the case of the Hubbard model the holons are noninteracting, the parameter $K = 1$, and the density of states behaves as $\nu^\pm(\varepsilon) \propto 1/\sqrt{\varepsilon}$, instead of the 3/8 power law (2).

To resolve this disagreement, we first notice that at $K = 1$ our Eqs. (7) and (8) are essentially equivalent to Eqs. (4), (10), and (13) of Ref. [11]. In the notations of Ref. [11] the functions (9) are given by $c_\sigma^+(q) = (N+1)C_{\sigma,N}(\pi - q)$ and $c_\sigma^-(q) = (N-1)D_{\sigma,N}(\pi - q)$ in the limit when the number of sites N in the spin chain is infinite. These functions have been computed numerically [11] by performing exact diagonalization of the Heisenberg spin chain (3) of up to 26 sites. At that system size the results have converged; they are shown schematically in Fig. 1. It is worth noting that the function $c_\sigma^+(q)$ is numerically small at $\pi/2 < |q| < \pi$, with $c_\sigma^+(\pm\pi) \approx 0.045$, whereas $c_\sigma^-(\pm\pi) \approx 0.46$ [11,16]. This indicates that at very low energies ε (but still $\varepsilon \gg J$) the peak (10) of the density of states is very asymmetric, with the tail below the Fermi level being higher than the one above it by an order of magnitude.

Given the numerical smallness of the leading asymptote (10) of the density of states at positive ε, it is worth considering the subleading contributions to the integral in Eq. (7). They come from the values of q in the range $-\pi/2 < q < \pi/2$, where $c_\sigma^+(q)$ is of order one. Taking into account the fact that $c_\sigma^+(q)$ diverges as $\chi/\sqrt{q + \pi/2}$ with $\chi \approx 0.8$ at $q \to -\pi/2 + 0$ [11,17,18], we find

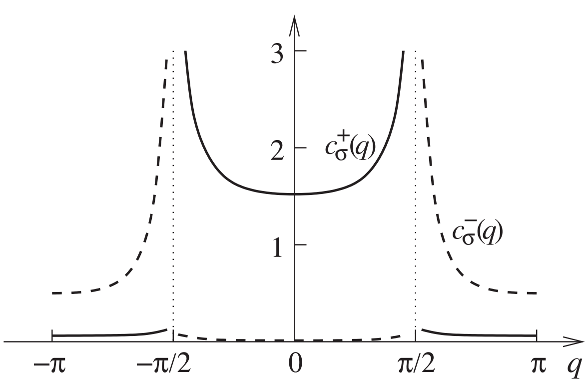

FIG. 1. Sketch of the dependences $c_\sigma^+(q)$ and $c_\sigma^-(q)$ at zero temperature (solid and dashed lines, respectively), based on the numerical results of Ref. [11].

$$\tilde\nu_\sigma^+(\varepsilon) = \nu_0 \frac{\chi}{\sqrt{2K}\Gamma(\frac{1}{2K} + \frac{K}{8})} \left(\frac{\varepsilon}{D_\rho}\right)^{(1/2K)+(K/8)-1} \frac{1}{\sqrt{\ln\frac{D_\rho}{\varepsilon}}}. \quad (11)$$

At $K = 1$ the exponent in Eq. (11) becomes $-3/8$, in agreement with Eq. (2).

At low positive energies the density of states can be treated as the sum of the contributions ν_σ^+ and $\tilde\nu_\sigma^+$, given by Eqs. (10) and (11). The subleading contribution $\tilde\nu_\sigma^+$ diverges less rapidly than ν_σ^+ at $\varepsilon/D_\rho \to 0$, but with the numerical coefficient that is larger by a factor of about 20. Thus for practical purposes the peak in the density of states $\nu_\sigma(\varepsilon)$ is given by $\tilde\nu_\sigma^+(\varepsilon)$, Eq. (11), at positive ε, and by $\nu_\sigma^-(\varepsilon)$, Eq. (10), at negative ε.

We now turn to the spin-incoherent regime, $T \gg J$. Assuming that $|\varepsilon| \gg T$, one can still use Eq. (7); however, the definitions (9) of the functions $c_\sigma^\pm(q)$ should now assume ensemble averaging. At $T \gg J$ the functions $c_\sigma^\pm(q)$ can be easily computed by using the following simple argument [19]. Given that the operators $Z_{l,\sigma}$ and $Z_{l,\sigma}^\dagger$ remove and add a site with spin σ at position l, it is clear that the ensemble average $\langle Z_{0,\sigma}^\dagger Z_{l,\sigma}\rangle$ equals the probability of all the spins on sites $0, 1, \ldots, l$ being σ. At $J \ll T$ the spins are completely random, so $\langle Z_{0,\sigma}^\dagger Z_{l,\sigma}\rangle = 1/2^{|l|+1}$. Similarly, $\langle Z_{l,\sigma} Z_{0,\sigma}^\dagger\rangle = 1/2^{|l|}$. Then the definitions (9) give

$$c_\sigma^+(q) = 2c_\sigma^-(q) = \frac{3}{5 - 4\cos q}. \quad (12)$$

The expression for $c_\sigma^-(q)$ is equivalent to the result for $D_{\sigma,N}(Q)$ found in Ref. [19].

It is important to point out that $c_\sigma^+(q)$ and $c_\sigma^-(q)$ differ by a factor of 2. As a result, the density of states (10) has a clear asymmetry: $\nu_\sigma(\varepsilon) = 2\nu_\sigma(-\varepsilon)$ at $T \ll \varepsilon \ll D_\rho$. The physical meaning of this result is very simple: the probabilities of adding an electron with spin σ at energy ε and removing one at $-\varepsilon$ differ by a factor of 2 because the electron that is being removed has the correct spin with probability 1/2.

The tunneling density of states can be studied experimentally by measuring the *I-V* characteristics of tunneling junctions in which one of the leads is a quantum wire. When the electron density in the wire is sufficiently low, the exchange coupling is expected to be exponentially weak, and the regime $J \sim T \ll D_\rho$ can be achieved [7]. In the experiment [20] such a measurement was performed in a situation where the second lead is another quantum wire. By applying magnetic field the authors have been able to observe momentum-resolved tunneling. To measure the density of states, it is more convenient to make a point junction from a metal tip to the side of the quantum wire. Contrary to the expectations based on the Luttinger-liquid theory, we predict that the *I-V* characteristic of such a junction will be very asymmetric with respect to reversal of the bias when $J \ll eV \ll D_\rho$. The dependence

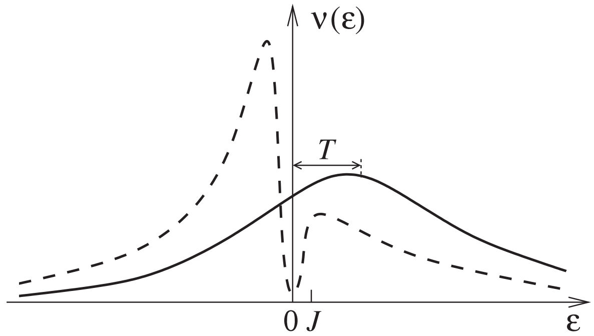

FIG. 2. Sketch of the tunneling density of states $\nu(\varepsilon)$ in two regimes: high temperature $T \gg J$ (solid line) and low temperature $T \ll J$ (dashed line). At $T \gg J$ and $\varepsilon \gg T$ we predict $\nu(\varepsilon) = 2\nu(-\varepsilon)$. As the temperature is lowered below the exchange constant J, the density of states at $\varepsilon < 0$ grows by about a factor of 3. Conversely, at $\varepsilon > 0$ the density of states decreases dramatically. At $|\varepsilon| \ll J$ the standard Luttinger-liquid effects give rise to power-law suppression of the density of states [2], leading to a dip at low ε.

$\nu_\sigma(eV) \propto dI/dV$ should have a peak at low bias at $K_\rho > 1/4$, or a dip at $K_\rho < 1/4$, but the asymmetry should be observed in either case.

The predicted asymmetry of the density of states is caused by the nontrivial interplay of the spin and charge degrees of freedom. As a result, the asymmetry should disappear in a polarizing magnetic field, $\mu_B B \gg T$, J. Indeed, if all spins are fixed in the $\uparrow$ direction, one easily finds $c_\uparrow^\pm(q) = 2\pi\delta(q)$. Then according to Eq. (7) the density of states $\nu_\uparrow(\varepsilon) \sim \nu_0(|\varepsilon|/D_\rho)^{\lambda(0)}$. This result recovers the standard Luttinger liquid suppression of the density of states [2] and shows no asymmetry around the Fermi level.

Conductance of quantum wires is expected to depend strongly on the ratio J/T [7]. Our theory provides a new way to probe this ratio by observing the asymmetry of the density of states $\nu(\varepsilon)$. The signature of the spin-incoherent regime, $J/T \ll 1$, is the doubling of the density of states at positive energies, compared to the negative ones, $\nu(\varepsilon) = 2\nu(-\varepsilon)$. At $J/T \gg 1$ the asymmetry is inverted, $\nu(-\varepsilon) > \nu(\varepsilon)$. This evolution of the density of states is described by Eq. (7), where the temperature dependence is contained in the functions $c_\sigma^\pm(q)$. Using Eq. (9) one can easily check that in the absence of magnetic field the integral of $c_\sigma^+(q)$ over all q is always larger than the integral of $c_\sigma^-(q)$ by a factor of 2. At high temperatures $c_\sigma^+(q) = 2c_\sigma^-(q)$, but as T becomes lower than J, the weight is redistributed so that $c_\sigma^+(q)$ is large at $|q| < \pi/2$, whereas $c_\sigma^-(q)$ is large at $|q| > \pi/2$, see Fig. 1. Since the expression (7) emphasizes larger values of $|q|$, the density of states $\nu(-\varepsilon)$ below the Fermi level increases, whereas $\nu(\varepsilon)$ decreases.

As the temperature is reduced, $c_\sigma^-(\pi)$ grows from $1/6$ at $T \gg J$, Eq. (12), to about 0.46 at $T/J \to 0$ [11,16]. Thus the density of states at negative energies will increase by nearly a factor of 3. At the same time the density of states at positive energies decreases. Because of the importance of the subleading contribution (11) the experimental results cannot be analyzed in terms of the behavior of $c_\sigma^+(\pi)$, Eq. (10). However, in the limit $T \to 0$, the density of states at sufficiently low positive energies will become much smaller than at the negative ones.

To summarize, we have established the true low-energy asymptote (10) of the density of states $\nu(\varepsilon)$ in a strongly interacting system of one-dimensional electrons. We predict $\nu(\varepsilon)$ to show asymmetric behavior illustrated in Fig. 2. The asymmetry is strongly temperature dependent even at $T \ll |\varepsilon|$. As the temperature is reduced, the system crosses over from the spin-incoherent regime at $T \gg J$ to the spin-coherent one at $T \ll J$, and the asymmetry changes sign. Although to the best of our knowledge no existing experiments probe the tunneling density of states in strongly interacting one-dimensional electron systems, we propose that a modification of the setup of Ref. [20] could be used for this purpose.

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