Conductance of Fully Equilibrated Quantum Wires

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We study the conductance of a quantum wire in the presence of weak electron-electron scattering. In a sufficiently long wire the scattering leads to full equilibration of the electron distribution function in the frame moving with the electric current. At nonzero temperature this equilibrium distribution differs from the one supplied by the leads. As a result the contact resistance increases, and the quantized conductance of the wire acquires a quadratic in temperature correction. The magnitude of the correction is found by analysis of the conservation laws of the system and does not depend on the details of the interaction mechanism responsible for equilibration.

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Experimental studies on the dc transport of short quantum wires have shown the quantization of their conductance in units of $2e^2/h$ [1]. This phenomenon is well understood within a model of noninteracting electrons [2], even though the interactions in the wire are usually not weak, i.e., $e^2/\hbar v_F \gtrsim 1$, where v_F is the Fermi velocity in the wire. The absence of any effect of electron interactions on the conductance is usually attributed to the fact that the quantum wires are always connected to two-dimensional leads, where interactions between electrons do not play a significant role. Indeed, it has been shown that within the so-called Luttinger-liquid theory, the interactions inside the wire do not affect conductance [3–5].

A number of recent experiments revealed deviations from perfect quantization in low-density wires [6–15]. These deviations often take the form of a shoulderlike feature, which develops at finite temperature just below the first quantized plateau, around $0.7(2e^2/h)$. At the moment, there is no consensus on the theoretical interpretation of this so-called "0.7 structure." It is generally accepted, however, that electron-electron interactions are involved in this feature, thus generating a lot of interest in understanding the effect of interactions on the transport properties of one-dimensional conductors. Here we study one of the most fundamental aspects of interactions, when they are so weak that their only effect is to equilibrate inside the wire the electrons coming from the two leads.

In the absence of interactions, the electrons propagate through the wire ballistically. Therefore the distribution functions of the right- and left-moving electrons are controlled by the left and right leads, respectively,

$$f_p = \frac{\theta(p)}{e^{(\epsilon_p-\mu_l)/T}+1} + \frac{\theta(-p)}{e^{(\epsilon_p-\mu_r)/T}+1}. \tag{1}$$

Here ϵ_p is the energy of an electron with momentum p, $\theta(p)$ is the unit step function, and we assume that the left and right leads have the same temperature T, but different chemical potentials $\mu_l = \mu + eV$ and $\mu_r = \mu$. It is important to note that even weak processes of electron-electron scattering will modify the distribution (1). Indeed, such processes will force some left-moving electrons to change their direction of motion and become right-movers. Thus the basic assumption of Eq. (1), that all the right-movers originate from the left lead and are in equilibium with it, will be violated.

The exact shape of the true steady state distribution of electrons can be understood easily if the wire is very long and the electron system is Galilean invariant, $\epsilon_p = p^2/2m$. In this case it is convenient to view the electron system in the reference frame moving with the drift velocity $v_d = I/ne$, where I is the electric current in the system and n is the electron density. In this frame the electron system is at rest, and must be described by the equilibrium Fermi-Dirac distribution characterized by a single chemical potential. In the stationary reference frame this distribution takes the form

$$f_p = \frac{1}{\exp(\frac{\epsilon_p - v_d p - \mu_{\rm eq}}{T_{\rm eq}})+1}, \tag{2}$$

where $\mu_{\rm eq}$ and $T_{\rm eq}$ approach μ and T at $V \to 0$. The distribution functions (1) and (2) coincide only in the limit of zero temperature. (In this case, to linear order in the drift velocity, $\mu_{l,r} = \mu_{\rm eq} \pm v_d p_F$, where p_F is the Fermi momentum.) At nonzero temperature, one can expect the full equilibration of the electron system in the wire to significantly affect its transport properties.

Recently, the equilibration of the electrons in the moving frame was shown to have a strong effect on the Coulomb drag between two parallel quantum wires [16] and to give rise to a finite resistivity of long inhomogeneous wires [17]. On the other hand, because the equilibration processes that relax the distribution (1) to the form (2) involve converting right-moving electrons into left-moving ones, it

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is natural to expect that the interactions will affect conductance of the wires even in the absence of inhomogeneities, Fig. 1. Such an effect was recently discussed by Lunde, Flensberg, and Glazman [18] in the case of short wires, where the effect of the equilibration processes is weak, and the distribution function is still close to the unperturbed form (1). They found a negative correction to the quantized conductance of the wire, which grows linearly with its length L.

In this Letter we explore the opposite limit of a long wire, $L \to \infty$, in which the electron distribution function in the wire does assume the limiting form (2), and the correction to the conductance saturates at a value independent of L. The crossover between this regime and that of short wires will be discussed elsewhere [19].

Throughout this Letter we assume that the interactions between electrons are very weak, and their only effect is to provide a mechanism of relaxation of the distribution function to the form (2). The exact nature of the scattering processes is unimportant, as long as these processes conserve the number of electrons, the energy of the system, and its momentum. Below we obtain the conductance of the wire by detailed analysis of these conservation laws.

The conservation of the number of particles means that in the steady state regime the total particle current $j(x)$ is constant along the wire. It is convenient to present the total current as the sum $j = j^R + j^L$ of currents of the right- and left-moving electrons,

$$j^{R,L}(x) = 2\int_{-\infty}^{\infty} \frac{dp}{h}\theta(\pm p)v_p f_p(x), \tag{3}$$

where the factor of 2 accounts for the spins, $v_p = p/m$ is the electron velocity, the positive sign in the step function corresponds to j^R, while the negative one to j^L.

It is important to realize that the distribution function f_p in Eq. (3) depends on the position in the wire. Inside a long wire, the relaxation processes ensure that f_p has the universal form (2), but near the ends of the wire f_p is affected by the leads. For example, at the left lead the distribution of the right-moving electrons is controlled by the lead and takes the form of the first term in Eq. (1). This enables one to easily evaluate the current $j^R(l)$ of the right-movers at the left lead.

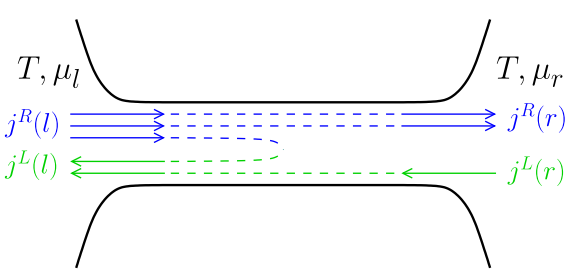

FIG. 1 (color online). Quantum wire in the regime of small applied bias, $\mu_l - \mu_r = eV$. The electric current is given by total currents of the right- and left-moving electrons, $I = e(j^R + j^L)$. The equilibration processes convert some of the right-moving electrons into the left-moving ones, thereby reducing the conductance of the wire.

Unlike the total current j, the current $j^R(x)$ is not uniform along the wire, as the equilibration processes allow electrons to change direction. The rate $\dot{N}^R$ of the change of the number of right-movers due to the electron-electron collisions is given by the difference of the values of j^R at the two ends of the wire, $\dot{N}^R = j^R(r) - j^R(l)$.

Although the current $j^R(r)$ of the outgoing right-movers is not known, it can be expressed in terms of the total current j and the current $j^L(r)$ of incoming left-movers, $j^R(r) = j - j^L(r)$. It follows then that the change in the number of right-movers per unit time $\dot{N}^R$ now depends on the electric current $I = ej$ flowing through the wire, as well as the sum of incoming particle currents from both leads

$$j^R(l) + j^L(r) = \frac{I}{e} - \dot{N}^R. \tag{4}$$

In analogy with $j^R(l)$, the current $j^L(r)$ is controlled by the right lead and can be found by using the distribution function f_p given by the second term in Eq. (1). Since both terms in the left-hand side of Eq. (4) are determined by the distribution functions in the noninteracting leads, the result of the routine evaluation of the two currents is given by the Landauer formula $j^R(l) + j^L(r) = 2eV/h$, up to corrections exponentially small in μ/T. Thus we find the following relation between the applied bias, electric current, and $\dot{N}^R$,

$$\frac{2e^2}{h}V = I - e\dot{N}^R. \tag{5}$$

An equivalent relation was obtained earlier in Ref. [18] using the Boltzmann equation formalism. It formally expresses the idea that the processes changing the number of right-movers in the wire will result in a correction to the quantized conductance.

The conservation of energy in electron-electron collisions implies that the total energy current $j_E(x)$ is uniform along the wire. It is instructive to express the energy current as the sum $j_E = j_E^R + j_E^L$ of the contributions of the right- and left-moving particles,

$$j_E^{R,L}(x) = 2\int_{-\infty}^{\infty} \frac{dp}{h}\theta(\pm p)v_p \epsilon_p f_p(x). \tag{6}$$

In the same fashion that we could relate the particle current j to how the number of right-moving electrons change over time, one can find a relation between the energy current j_E flowing through the wire and the rate of change $\dot{E}^R$ of the energy of right-movers due to the electron collisions. Indeed, the reasoning that led to Eq. (4) can be readily extended to the case of energy currents rather than particle ones, leading to

$$j_E^R(l) + j_E^L(r) = j_E - \dot{E}^R. \tag{7}$$

The energy currents in the left-hand side of Eq. (7) are again controlled by the leads and can be easily computed using the distribution function (1) of noninteracting electrons. At low temperatures $T \ll \mu$ one finds $j_E^R(l) + j_E^L(r) = (2eV/h)\mu$, up to corrections as small as $e^{-\mu/T}$.

Since the energy current j_E does not depend on the position in the wire, one can calculate it in a region away from the leads, where the distribution function is given by Eq. (2). The formal calculation gives $j_E = \mu(1 + \pi^2 T_{\rm eq}^2/6\mu_{\rm eq}^2)j$, where we discarded terms of order $O((T_{\rm eq}/\mu_{\rm eq})^4)$ and higher. This result can be compared with the above-mentioned calculation for the unperturbed distribution (1), which can be summarized as $j_E = \mu j$. The difference can be traced back to the energy dependence of the term $v_d p$ in the Fermi function (2). Expressing the particle current as $j = I/e$ we then find

$$\frac{2e^2}{h} V = \left[1 + \frac{\pi^2}{6}\left(\frac{T}{\mu}\right)^2\right] I - \frac{e}{\mu}\dot{E}^R. \tag{8}$$

This result is obtained in the linear order in the applied bias, which enabled us to substitute $T_{\rm eq} = T$ and $\mu_{\rm eq} = \mu$.

In the absence of the scattering processes changing the number of the right- and left-moving electrons, not only $\dot{N}^R$, but also $\dot{E}^R$ would vanish. Indeed, in this case the two branches of excitations would represent two electron systems with no particle exchange allowed. Then the distribution function (1) would describe the systems of right- and left-movers in thermal equilibrium with each other, and the net heat transfer between them would vanish. Thus both $\dot{N}^R$ and $\dot{E}^R$ arise as a consequence of the same relaxation mechanism inside the wire, and we will now show that there is a simple relation between these rates.

In the case of a short wire it was shown [18] that the dominant process changing the number of right-movers involves three electrons, with a right-mover near the bottom of the band reducing its momentum and thus the direction of motion, Fig. 2(a). The conservation of momentum then requires that the other two electrons increase their momenta. Finally, conservation of energy requires one of these two electrons to be near the right Fermi point, and the other near the left one. The typical momentum change is controlled by the temperature, $|\delta p| \sim T/v_F$. As a result of such scattering events the distribution function (1) shows only a small modification whereby the exponentially small discontinuity near $p = 0$ is smeared.

A much more significant change occurs in longer wires, where the relaxation processes bring the distribution function to the form (2). A comparison of the distribution functions (1) and (2) shows that the main difference between them is at the values of momentum p near the Fermi points $\pm p_F$. Thus the dominant relaxation processes contributing to $\dot{N}^R$ take electrons with $p \approx p_F$ and move them to $p \approx -p_F$. Such processes are realized in many small steps of $|\delta p| \sim T/v_F$ and are accompanied by multiple electrons scattering near the two Fermi points, see Fig. 2(b). The total momentum transferred to these electrons is $2p_F$. Energy conservation requires that it is distributed evenly between the right- and left-movers, so that the resulting energy increase $\delta E^R = v_F p_F$ is compensated by the decrease $\delta E^L = -v_F p_F$.

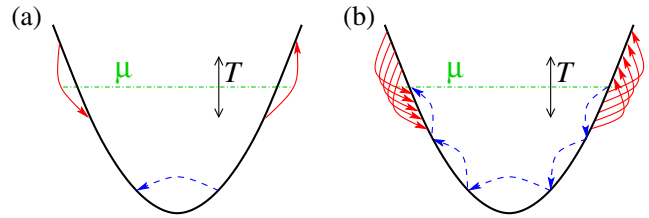

FIG. 2 (color online). Illustration of the relaxation processes converting a right-moving electron into a left-moving one. The parabolas represent the quadratic dispersion $\epsilon_p = p^2/2m$ of the electrons in the wire. (a) A three-particle process studied in Ref. [18]. Momentum and energy conservation ensure that as a right-mover changes direction, another right-mover increases its energy and a left-mover decreases it. (b) Transfer of an electron from the right to the left Fermi point is accompanied by multiple right-movers increasing their energies and multiple left-movers decreasing energies.

In the end, the energy balance for the right-moving electrons consists of a loss of ϵ_F, which was the energy of the electron changing direction, and a gain of $v_F p_F = 2\epsilon_F$ due to the redistribution of momentum. As a result, for every right-moving electron that changes direction, $\Delta N^R = -1$, the right-movers' energy increases by an amount $\Delta E^R = \epsilon_F$. We thus conclude that

$$\dot{E}^R = -\mu \dot{N}^R, \tag{9}$$

where we replaced ϵ_F with μ, as the small difference $\mu - \epsilon_F \sim T^2/\mu$ turns out to be irrelevant for our purposes. It is important to note that the result (9) is not sensitive to the specific details of the electron relaxation mechanism. Indeed, the two key ingredients of this derivation are the conservation laws that control the redistribution of momentum $2p_F$ between the right- and left-movers, and the quadratic dispersion that governs how the energies of the two subsystems change as a result of that redistribution.

By analyzing the conservation laws we have so far been able to establish three linear relations (5), (8), and (9) between four quantities, V, I, $\dot{N}^R$, and $\dot{E}^R$. Assuming that the applied bias V is known, we can now express the remaining three quantities in terms of V. Most importantly, we find $I = GV$, with the conductance

$$G = \frac{2e^2}{h}\left[1 - \frac{\pi^2}{12}\left(\frac{T}{\mu}\right)^2\right], \tag{10}$$

where we restricted ourselves to the leading order term in $(T/\mu)^2$.

The quadratic in temperature correction to the quantized conductance $2e^2/h$ of the wire is our main result. Unlike the correction to the conductance of a short wire $\delta G \propto e^{-\mu/T}$ [18], our correction shows power-law dependence on the temperature. Earlier papers [3–5] on the conductance of long quantum wires did not find any correction to the conductance, as the Luttinger-liquid theory used there does not account for the relaxation processes leading to our result (10).

Experimentally, small temperature-dependent corrections to quantized conductance have been observed in quantum point contacts [6–9]. The latter are essentially short quantum wires, with only a few electrons in the one-dimensional part of the device. In order for our result (10) to be fully applicable the length of the system should be sufficient to ensure full equilibration of the electron distribution function. Comparison of our correction $\delta G \sim (T/\mu)^2$ with the result [18] for short wires, $\delta G \propto Le^{-\mu/T}$ implies that our result is applicable at $L \gg l_{\mathrm{eq}} \propto e^{\mu/T}$, in agreement with the more detailed calculation [19]. Although some experiments with longer quantum wires have been reported [10,12], a careful study of the temperature-dependent corrections to the conductance is not yet available.

In summary, we have shown that in a long quantum wire, the full equilibration of the electron distribution function leads to a finite correction to the conductance, which at $T \ll \mu$ is quadratic in temperature, Eq. (10). Our derivation relied uniquely on an analysis of the conservation laws for energy, momentum, and particle number, without making specific assumptions regarding the process of equilibration.

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