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Multiple-phonon-assisted tunneling in a magnetic field

A. V. Khaetskii*
Max-Planck-Institut für Festkorperforschung, Heisenbergstrasse 1, D-7000 Stutgart 80, Germany

K. A. Matveev
Institute of Solid State Physics, U.S.S.R. Academy of Sciences, 142432, Chernogolovka, Moscow district, U.S.S.R.
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Electron tunneling in transverse magnetic fields is studied with regard to multiple-subbarrier scattering by phonons. It is shown that the electron-phonon interaction facilitates tunneling considerably because of the shift of the magnetic-oscillator center upon scattering. This involves, in particular, a strong temperature dependence of the tunnel-junction conductance in magnetic fields parallel to the dielectric-interlayer plane. These effects may show up in the hopping conductivity in a magnetic field.

Electron tunneling is known to be strongly suppressed by a transverse magnetic field. In particular, in the presence of a field H directed along the z axis the asymptotic behavior $\exp(-r/a)$ of the wave function of the ground state of an impurity-bound electron is substituted by

$$\psi(\rho,z)\propto\exp\left[-\frac{|z|}{a}-\frac{\rho^2}{4\lambda^2}\right], \tag{1}$$

where $\lambda=(c\hbar/eH)^{1/2}$ is the magnetic length and $\rho=(x,y)$. Such a change in the asymptotic behavior of the wave function may be accounted for by the occurrence of an additional potential barrier $\hbar^2\rho^2/8m\lambda^4$ called a magnetic barrier.[1] In Refs. 2–4 it has been shown that if the electron is scattered by impurities (or the crystal boundary) in the course of tunneling, the influence of the magnetic field on tunneling is considerably moderated: the asymptotic behavior of the wave function takes the form

$$\psi\propto\exp(-\rho/b), \tag{2}$$

where b is the characteristic length dependent on H and on the scattering intensity. This result may be interpreted in the following way. In each scattering act the electron transfers a momentum to the impurity in the direction perpendicular to that of tunneling, shifting the center of the magnetic oscillator in the tunneling direction. With further movement the magnetic barrier is centered on the scattering point. As a result, upon multiple scattering the magnetic barrier ceases to increase monotonically with increasing ρ, which leads to dependence (2).

The present work is concerned with the case when subbarrier scattering on impurities may be neglected and the oscillator center is shifted due to transfer of momentum to phonons. Unlike scattering on static defects,[2–4] the transfer of momentum to phonons is inevitably accompanied by energy transfer. Therefore, the effects in question prove to be important in the cases when the processes of inelastic electron tunneling are prevailing, for instance, in hopping conductivity in semiconductors[1] or in tunnel junction conductivity at not too low temperatures.

For definiteness, let us consider a tunnel junction in a magnetic field parallel to the dielectric-interlayer plane (see Fig. 1). In contrast to Ref. 3, it is assumed that the interlayer is free from impurities. On the contrary, the leads are assumed to comprise high concentrations of impurities so that the Landau quantization in them may be neglected. We consider the case of $\lambda\ll d$ when the tunneling conductivity of the junction is substantially suppressed by the magnetic field (d is interlayer thickness). The temperature dependence of linear conductance will be calculated. It will be shown that the processes of emission and absorption of phonons are initiated at some threshold temperature, which leads to a decrease of the effective magnetic barrier and, hence, an exponential increase of conductance with temperature.

We will calculate the conductance in the linear regime $V\ll T/e$ by the formula

$$G=\frac{e^2}{T}\sum_{k,p}W_{kp}n_k(1-n_p). \tag{3}$$

Here indices k and p number the states in the left and right junction leads, respectively, n_k and n_p are the Fermi filling functions of the states in the leads; W_{kp} is the probability of tunneling from state k to state p per unit time. We can obtain formula (3) proceeding from the expression for the current

$$I=e\sum_{k,p}[W_{kp}n_k(1-n_p)-W_{pk}n_p(1-n_k)],$$

where $n_k=n_F(E_k-eV)$, $n_p=n_F(E_p)$. Then, using the relation between the probabilities of forward and backward

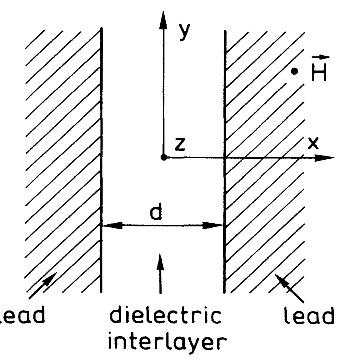

FIG. 1. Tunneling geometry.

transition,

$$W_{kp}=W_{pk}\exp\left(\frac{E_k-E_p}{T}\right),$$

for the quantity $G=dI/dV|_{V=0}$, we obtain expression (3). The probability W_{kp} may be written as

$$W_{kp}=\sum_{n=0}^{\infty}W_{kp}^{(n)}=\sum_{n=0}^{\infty}\alpha^{2n}w_{kp}^{(n)}. \tag{4}$$

Series (4) is an expansion in terms of the constant α of the electron-phonon interaction ($\alpha \ll 1$). Further probabilities $W_{kp}^{(n)}$ will be calculated within exponential accuracy.

First, let us discuss the initial term of the series $W_{kp}^{(0)}$ which is important within the limit $\alpha \rightarrow 0$ or, as we shall see below, at sufficiently low temperatures. It describes elastic tunneling of electrons from one lead to the other and is proportional to the square of the modulus of the transition-matrix element $T_{kp}^{(0)}$. In the absence of impurities k (or p) may be thought of as the coordinate of the magnetic-oscillator center which is conserved and, hence, $T_{kp}^{(0)}=0$. Consequently, the presence of impurities in the leads is essential for current to pass through the junction. It may be shown that given impurities in the leads, the dependence of the elastic conductance $G^{(0)}$ on interlayer thickness takes the form

$$G^{(0)}=\beta_1 n\exp(-d^2/\lambda^2)+\beta_2 n^2\exp(-d^2/2\lambda^2). \tag{5}$$

Here n is the two-dimensional (2D) concentration of impurities in the narrow layer of thickness λ^2/d adjacent to the barrier, and the explicit form of the coefficients β_1,β_2 is not important for us. Equation (5) may be understood in the following way. The first term in (5) describes, for instance, the electron scattering from the left lead to the right one on the right-lead impurities. The corresponding matrix element of the transition is proportional to the value of the wave function of the electron in the left lead on the opposite side of the barrier, $T_{kp}^{(0)}\propto\psi_k(d)\propto\exp(-d^2/2\lambda^2)$. The initial term in (5) is proportional to the square of this quantity. The second term in (5) corresponds to electron scattering by a pair of impurities located at points r and r' in different leads. The matrix element of this transition

$$\sum_m\frac{V_{km}V_{mp}}{E_k-E_m}\propto\int d^3r_1\,d^3r_1'\,\Psi_k^*(\mathbf{r}_1)v(\mathbf{r}_1-\mathbf{r})g(\mathbf{r}_1,\mathbf{r}_1')v(\mathbf{r}_1'-\mathbf{r}')\Psi_p(\mathbf{r}_1')\propto\Psi_k^*(\mathbf{r})\Psi_p(\mathbf{r}')g(\mathbf{r},\mathbf{r}')$$

[$v(r)$ is the impurity potential] appears to be proportional to the Green function $g(\mathbf{r},\mathbf{r}')\propto\exp(-|\mathbf{r}-\mathbf{r}'|^2/4\lambda^2)$. Further, we shall assume that the impurity concentration is not exponentially small so that the conductance is determined by the second term in (5). Thus, as in Ref. 3, the elastic conductance is proportional to the square of the modulus of the Green function along the shortest way.

This result may be generalized to the case of tunneling with emission or absorption of phonons. Specifically, the amplitude of electron tunneling from point $\mathbf{r}$ in the left lead to point $\mathbf{r}'$ in the right lead with absorption of phonons with wave vectors $\mathbf{q}_1,\ldots,\mathbf{q}_m$ and emission of phonons $\mathbf{q}_1',\ldots,\mathbf{q}_l'$ is proportional to the matrix element of the operator Green function

$$\langle\mathbf{r}';(N_{\mathbf{q}_1}-1)\cdots(N_{\mathbf{q}_m}-1),(N_{\mathbf{q}_1'}+1)\cdots(N_{\mathbf{q}_l'}+1)|\frac{1}{E_k-\hat{H}}|\mathbf{r};N_{q_1},\ldots,N_{\mathbf{q}_m},N_{\mathbf{q}_1'},\ldots,N_{\mathbf{q}_l'}\rangle. \tag{5a}$$

Here $\hat{H}$ is the Hamiltonian of the system including the electron-phonon interaction. The matrix element (5a) is calculated by expanding of the operator $1/(E_k-\hat{H})$ in a power series in the electron-phonon-interaction operator. The expression for probability (4) so obtained has the form

$$W_{kp}\propto\sum_{\mathbf{r},\mathbf{r}'}|\psi_k(\mathbf{r})|^2|\psi_p(\mathbf{r}')|^2\sum_{m,l}\alpha^{2(m+l)}\sum_{\{\mathbf{q},\mathbf{q}'\}}\exp\left[-\sum_{i=1}^{m}\frac{\lambda^2q_i^2}{2}-\sum_{j=1}^{l}\frac{\lambda^2q_j'^2}{2}-\frac{1}{2\lambda^2}[(x-x'+\lambda^2Q_y)^2+(y-y'-\lambda^2Q_x)^2]-\frac{E_{\text{abs}}}{T}\right]\delta(E_k-E_p+E_{\text{abs}}-E_{em}). \tag{6}$$

Here the sum over $\mathbf{r}$ and $\mathbf{r}'$ is taken over the positions of impurities in the left and right leads, respectively, m (l) being the number of absorbed (emitted) phonons, $\hbar\mathbf{Q}$ the full momentum transferred to the phonons,

$$\mathbf{Q}=\sum_{j=1}^{l}\mathbf{q}_j'-\sum_{i=1}^{m}\mathbf{q}_i.$$

The energies E_{abs} and E_{em} entering in (6) are the total energies of absorbed and emitted acoustic phonons,

$$E_{\text{abs}}=\sum_{i=1}^{m}\hbar s|\mathbf{q}_i|,\quad E_{\text{em}}=\sum_{j=1}^{l}\hbar s|\mathbf{q}_j'|,$$

and in (6) it is assumed that all the phonon energies exceed the temperature. In Eq. (6) it has been taken into account that phonons with small q_z turn out to be most important in the tunneling process.

In order to make clear the sense of formula (6), consider the term of the sum with $m=1$, $l=0$ corresponding to tunneling with the absorption of one phonon. In this case the amplitude of electron tunneling from point $\mathbf{r}$ to point $\mathbf{r}'$, calculated by using expression (5a), is proportional to the following quantity:

$$\int d^3r_1 g(\mathbf{r},\mathbf{r}_1)\exp(-i\mathbf{q}_1\mathbf{r}_1)g(\mathbf{r}_1,\mathbf{r}').$$

The term in question of the sum in (6) is proportional to the square of the modulus of this quantity. Since the terms of the sum in (6) decay quickly with increasing $|\mathbf{r}-\mathbf{r}'|$, we assume that $x'-x=d$, $y'-y=0$ (the x axis is directed along the normal to the interlayer plane). If the difference E_p-E_k is not very large ($E_p-E_k<\hbar sd/2\lambda^2$), the greatest contribution is made by phonons with wave vectors directed along the y axis. Hence, for the probability $W_{kp}^{(1)}$ we obtain

$$W_{kp}^{(1)} \propto \alpha^2 \exp\left[-\frac{x_0^2}{2\lambda^2}-\frac{(d-x_0)^2}{2\lambda^2}\right]\exp\left[-\frac{E_p-E_k}{T}\right],$$

$$E_p>E_k,\ x_0=\lambda^2 q_1=\lambda^2\frac{E_p-E_k}{\hbar s}. \tag{7}$$

The value x_0 is the shift of the magnetic-oscillator center due to absorption of a phonon with momentum $\hbar q_1$. Contrary to the case of static defects, the momentum exceeding $|E_p-E_k|/s$ cannot be transferred to the phonon. Owing to this, the shift of the oscillator center cannot exceed x_0.

Using formulas (3) and (6), we can calculate the conductance to any order in the electron-phonon-interaction constant to within exponential accuracy. At exponentially small α the main contribution to the temperature dependence of conductance is made by single-phonon processes which may be considered with the aid of Eq. (7),

$$G^{(1)} \propto \begin{cases} \alpha^2\exp(-d^2/2\lambda^2), & L_T>d, \\ \alpha^2\exp\left[-\frac{d^2}{2\lambda^2}+\frac{(d-L_T)^2}{4\lambda^2}\right], & L_T<d. \end{cases} \tag{8}$$

The length L_T introduced here is defined by sound velocity and temperature,

$$L_T\equiv \hbar s/T. \tag{9}$$

As seen from (8), there exists a threshold temperature

$$T_1=\hbar s/d.$$

From this temperature on, correction (8) increases exponentially. As seen from Eq. (7), for $T<T_1$ the tunneling processes with a shift of the oscillator center are suppressed because of the small probability of finding the phonon required. At high temperatures $T\gg T_1$, when this probability is not small, the tunneling processes that shift the oscillator center by the value $x_0=d/2$ are dominant. As would be expected in this case, $G^{(1)}\propto\exp(-d^2/4\lambda^2)$.

As with scattering on impurities, at not too small α the multiple-phonon contributions to conductance appear to be exponentially greater than the single-phonon one owing to more efficient suppression of the magnetic barrier. In particular, the double-phonon contribution has the form

$$G^{(2)} \propto \begin{cases} \alpha^4\exp(-d^2/2\lambda^2), & L_T>2d, \\ \alpha^4\exp\left[-\frac{d^2}{2\lambda^2}+\frac{(d-L_T/2)^2}{3\lambda^2}\right], & L_T<2d. \end{cases} \tag{10}$$

Indeed, at high temperatures (when $L_T\ll d$) we have $G^{(2)}\propto\exp(-d^2/6\lambda^2)$. Note that the exponential increase in $G^{(2)}(T)$ starts from the threshold temperature

$$T_2=\frac{\hbar s}{2d}. \tag{11}$$

Therefore, the threshold temperature for the double-phonon processes is half as much as that for single-phonon ones. This may be understood in the following way. Compare the single- and double-phonon processes that involve the same full shift Δx of the oscillator center in the direction of tunneling and, hence, the same full momentum transferred to phonons. The contribution of the process with single-phonon absorption contains the small Boltzmann multiplier $\exp(-\Delta E/T)$, where $\Delta E=(\hbar s)\Delta x/\lambda^2$, which is related to the small probability of finding a phonon with the required momentum. (For phonon emission the same multiplier occurs because of the small number of vacant states in the right lead at energy $E_F-\Delta E$.) For double-phonon processes this multiplier has the form $\exp(-\Delta E/2T)$, since the possibility exists of transferring one half of the required momentum to the absorbed phonon and the other half to the emitted phonon. (For the processes with the absorption of two phonons or the emission of two phonons, the Boltzmann multiplier is the same as for the single-phonon processes).

Thus, at realistic values of α the low-temperature behavior conductance is determined by the double-phonon processes rather than single-phonon ones and is described by formula (10). Further increase in temperature initiates in succession the processes involving $3,4,5,\ldots,$ phonons. When the optimal number of phonons is large, we found that the temperature dependence of conductance is defined by the expression

$$G\propto\exp\left[-\frac{d}{\lambda}\left(\frac{L_T}{2\lambda}+2\ln^{1/2}(1/\alpha)\right)+\frac{L_T^2}{8\lambda^2}+\frac{L_T}{\lambda}\ln^{1/2}(1/\alpha)\right]. \tag{12}$$

Note that by virtue of multiphonon scattering, the exponent of (12) is free of the term quadratic in d/λ.

In conclusion it should be noted that the mechanism of multiple-phonon tunneling under consideration may be important for describing hopping conductivity in a magnetic field. As in the case of nonresonant impurity scattering[2] phonon scattering leads to the linear dependence of the exponent of the probability of a single hop on its length. At sufficiently high temperature, when L_T is smaller than the mean impurity spacing, hopping conductivity in a magnetic field is likely to be determined by the multiple-phonon processes.

*Institute of Microelectronics Technology and High Purity Materials, U.S.S.R. Academy of Sciences, 142432, Chernogolovka, Moscow district, U.S.S.R.

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